Competition for Flow and Short-Termism in Activism*

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Abstract

We develop a dual-layered agency model to study blockholder monitoring by activist funds that compete for investor flow. Competition for flow affects the manner in which activist funds govern as blockholders. In particular, funds inflate short-term performance by increasing payouts financed by higher (net) leverage, which subsequently discourages value-creating interventions in economic downturns due to debt overhang. Our theory suggests a new channel via which asset manager incentives may foster economic fragility and links together the observed procyclicality of activist investments with the documented effect of such funds on the leverage of their target companies.

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Activist blockholders play a key role in mitigating governance problems in publicly traded corporations with dispersed owners who have limited incentives to monitor managers. The potential benefits of blockholders have been widely recognized in the theoretical literature on corporate governance since Shleifer and Vishny (1986). In recent decades institutional investors such as hedge funds and private equity funds have taken the lead in shareholder activism (Gillan and Starks 2007). It is important to recognize that—unlike the blockholders of classical corporate governance models—such institutional activists are delegated portfolio managers who rely on the approval of the investors who finance them. In particular, funds must compete via performance to retain and attract investor capital, commonly referred to as competition for flow.[1]

In this paper we develop a dual-layered agency model to study blockholder monitoring by activist funds who compete for flow. Funds are principals as active owners in target firms, who tackle a managerial agency problem and enhance target firm value by intervention. Simultaneously, funds are agents who manage portfolios for clients and must compete for flow. We show that such competition for flow affects how activist funds govern as blockholders, fostering short-termism and reducing the efficacy of activism. In particular, competition induces them to inflate short-term fund performance by increasing payouts, financed by higher (net) target firm leverage. This, in turn, discourages value-creating interventions in economic downturns due to debt overhang.

While our paper primarily examines the role of institutional investors in corporate governance, it also offers a new perspective on systemic risks arising from asset managers' incentives. Following the recent deleveraging in the banking sector, several commentators call for attention to be shifted to incentives and competition in the asset management industry as a potential source of financial instability (e.g., the International Monetary Fund 2015 Financial Stability Report, Chapter 3). Recent academic work in response to such concerns has focussed on fire sales of financial assets triggered by redemption

[1]Rewards for performance have been documented across many classes of institutional investors, e.g., Chung et al. (2012) for private equity and Lim et al. (2016) for hedge funds.

threat as a source of systemic risk (e.g., Morris and Shin 2016; Zheng 2017). In contrast, our model shows how competition for flow amongst activist funds can foster excessive leverage in target firms which acts as a source of fragility by exacerbating procyclicality. We thus provide a complementary perspective by demonstrating how fund manager incentives can impact leverage and investment in the real economy.

The key elements of our model can be summarized as follows. Activist funds own blocks in target firms, tackle managerial agency problems, and intervene to raise target firm value. Some of these activities are feasible in the short-term (e.g., releasing excess cash from target firms) while others take time and extended effort to implement (e.g., business improvements, restructuring, or merger of the target). The potential returns to longer-term activism are exposed to changes in economic conditions (e.g., takeover premia are sensitive to aggregate economic conditions). Activist funds differ in their intrinsic ability to generate returns: Good funds are able to generate higher cash flows from each form of activism than bad ones. Funding for activists is provided by their fee-paying investors to whom the funds provide periodic returns. These investors make (rational) inferences about the ability of their funds based on these returns, and then decide whether to take their money elsewhere.

While tackling managerial agency problems, the need to compete to keep investor capital tempts funds to enhance their intrinsically generated returns. They do so by surreptitiously moving resources forward in time, i.e., by borrowing today against the target firm's future cash flows. Investors, in turn, are fully capable of detecting and nullifying such enhancement activity by incurring a small verification cost. We impose a small verification cost because the financing of target firms is arguably not fully transparent (in real time) to fund investors.

We model short-termism triggered by competition for flow as excessive leverage, but could have chosen some other strategy that boosts current earnings at the expense of long-term profitability. An example of such a strategy is cutting target firm R&D

expenditure which—as in our model—inflates current returns to fund investors at the expense of long-term target cash flows. More generally, leverage in our model can be thought of as a metaphor for any action that an activist fund may take that boost short term payouts at the expense of long-term prospects for the target firm.

Returning to our model, we now provide a roadmap for our main results. We first show that there is no pooling equilibrium in which both types of funds make identical payouts. If bad funds were to successfully enhance their early returns in an attempt to pool with the good, investors would prefer to verify and thereby nullifying the mimicking attempt. Thus, in any feasible equilibrium good funds lever the target firm to enhance payouts and thus separate from bad funds.

In our first core result, we characterize conditions under which—even in the separating equilibrium with the *minimal* amount of leverage that can support separation—borrowing is high enough to generate debt overhang in low aggregate states leading to a shutdown in activist effort (Proposition 1). Thus activism becomes *fragile*. Importantly, such fragility—which underpins all subsequent results—can only arise in the presence of competition for flow: absent competition, there is no need to boost intrinsic performance by borrowing, and thus no debt overhang.

Our second core result (Proposition 2), delineates the role of aggregate economic prospects in affecting the fragility of activism, its profitability, and target firm leverage. In particular, higher economic prospects make activism more fragile, increase returns from investing in activist funds, and increase target firm leverage.

For expositional ease, we present our theoretical analysis in two steps. The results summarized above are first derived in a simplified setting (developed in Section 1) in which financing and compensation contracts are taken as given: activists use target firm debt to boost short-term performance, and investors compensate activists via a given contract that is motivated by real world money management contracts. Subsequently, in Section 4, we endogenize these contract choices in an enriched setting featuring a single contractual friction – the nonverifiability of aggregate economic states. In this setting, we show that debt is the optimal contract for raising external financing and no compensation contract for fund managers dominates the previously assumed one.

Our paper is theoretical, but it offers several points of contact with observed activism by leading classes of institutional investors. One such class is activist hedge funds. These funds have taken centre stage in activism (e.g., Gillan and Starks 2007), generating gains to target firms in terms of share prices and operating performance (see the survey by Brav, Jiang, and Kim 2010). Two key themes emerge from our analysis. First, since activist funds enhance payouts via increased net leverage, target firms experience increases in payout and leverage. Second, as a result of the procyclicality discussed above, investment in activist funds are higher in bull markets. Both implications resonate with the available empirical evidence on activist hedge funds, as discussed in Section 3. In that section, we also extend our baseline analysis, to deliver results that are geared towards specific findings and debates in the empirical literature on activist hedge funds.

Our model also sheds light on the broader debate at to whether hedge fund activism creates value or not. While some legal scholars and commentators (see Kahan and Rock 2007 for an overview) argue that hedge fund activists are short-termist and destroy firm value, motivated by the above evidence we make modeling choices that ensure that activism is overall beneficial to target firm value. Simultaneously, our model highlights that activism also comes with endogenously generated costs: due to competition for flow, activists are short-termist and amplify the exposure of target firms to aggregate economic fluctuations, fostering fragility, and limiting the efficacy of activism.

Another class of institutional investors pertinent to our model is private equity. It is often argued that the buyout activity of private equity funds is procyclical.[2] Further, the use of extensive leverage in private equity buyouts is well known. Thus, at a qualitative

[2]In a model of the optimal financing structure of private equity funds, Axelson, Stromberg, and Weisbach (2009) demonstrate how the procyclicality of funding implies overinvestment in booms and underinvestment in busts.

level, our debt overhang story provides an explanation for the cyclical features of private equity buyout activity as well. Indeed, consistent with our results in Proposition 2, Axelson et al. (2013) find that private equity buyout leverage is procyclical.[3]

Our paper belongs to the large literature on blockholder monitoring — active monitoring via "voice" or passive monitoring via "exit" — in publicly traded corporations (see Edmans and Holderness (2017) for a survey). This literature abstracts from the delegated nature of blockholding, a phenomenon particularly prominent in the US and the UK, but also relevant elsewhere. By contrast, we focus on delegated blockholders and model how competition for flow affects how they govern via voice. A few recent papers have started to explicitly consider the impact of the incentives of fund managers on blockholder monitoring. Song (2017) considers multiple blockholders who govern via voice and exit. While we show how competition for flow can generate debt overhang, he argues that such competition can be beneficial when there are multiple heterogeneous blockholders. This is because Song models fund managers as stock pickers (i.e., non-activist funds). Such fund managers are reluctant to intervene in a company in which they hold a position because the need to intervene may reveal their poor stock selection. The presence of such a blockholder who is reluctant to intervene catalyzes other blockholders (with longer horizons) to intervene. In contrast, our fund managers are specialists in activism and thus intervention per se is never a negative signal. Song's model builds on Dasgupta and Piacentino (2015) who, like us, consider the negative impact of microfounded flow motivations but, in constrast to us, focus on passive monitoring via exit. Similarly Goldman and Strobl (2013) also differ from us by examining passive monitoring, and further, also assume that delegated blockholders have short horizons. Their focus — on the interplay of horizon mismatch, stock price manipulation, and investment complexity — is very different from ours.

Last but not least, our model builds on the insights of classical corporate finance

[3]Two recent theoretical papers that examine specifically the procyclicality of private equity buyout activity are Martos-Vila, Rhodes-Kropf, and Harford (2019) and Malenko and Malenko (2015).

papers on debt overhang (Myers 1977) and dividend signalling (e.g., Bhattacharya 1979; Miller and Rock 1985).[4] As in the latter strand, payouts act as a signal; such payouts are financed via debt, leading to overhang as in the former strand. The distinctive feature of our paper is the governance problem as the incentive for signalling. In a classical governance model, if managers' short-termism leads to excessive payouts, a large shareholder should attempt to curtail these. In our setting, corporate managers are not short-termist. The source of agency conflicts at the firm level is a free cash flow problem. However, it is the large shareholders who are endogenously short-termist because they are funds that compete for flow. As a result, our large shareholders can solve the firm-level free cash flow problem, but at the cost of paying out excessively and fostering debt overhang. This is driven by the dual layered nature of the agency problem: the fund is the principal with respect to the firm but simultaneously an agent with respect to their investors.

1 Model

In our model *activist funds* are involved in two agency relationships, as principals in one and as agents in the other. On the one hand, funds are active owners (principals) in *target* firms who increase firm value by tackling a managerial agency problem and by contributing their expertise. On the other hand, funds are delegated portfolio managers (agents) financed by *investors* (IN) who pay fees to them and evaluate their performance. In addition, there are competitive, deep-pocketed *financiers* (FI) who may provide financing to firms targeted by funds.

There are two periods ($t = 1, 2$), and many firms, funds, investors, and financiers. Each fund is financed by an investor and enters the first period having used the investor's capital to acquire a stake in a target firm. We assume that each fund is in de facto control

[4] Another classical contribution to the latter strand is by Ross (1977), in which (unlike in our model) external financing is itself a signalling device.

of its target firm. In other words, we do not model the phenomenon by which activist funds are able to obtain decisive influence in target firms.[5] Accordingly, we assume for simplicity that each fund owns all shares of its target firm. Each target firm can subsequently borrow funds from a financier. All actors are risk-neutral and there is no discounting.

Activism. Activist funds come in two types $\theta \in \{G, B\}$, where $\Pr(\theta = G) = \gamma_\theta$. Regardless of type funds can engage in two forms of activism, each of which increases target firm cash flows. The first form of activism can be implemented relatively quickly while the second takes time and effort. For concreteness, we consider specific manifestations of these two types of activism. In the short run, activists ameliorate a free-cash flow problem in the target firm. In the long run, activists add value by contributing their expertise to a range of activities that we collectively term restructuring. Furthermore, the model can be more broadly interpreted, as we discuss in Section 3.4.

Short-term activism (t = 1). Short-term activism addresses a free cash flow problem in the target firm. Each target firm has an amount of cash $C > 0$ and is run by an empire building manager. If left under the manager's discretion, C will be invested in wasteful projects. For simplicity, these wasteful projects are assumed to have zero return. Funds are differentially skilled in identifying potential projects as being wasteful and can thus salvage a type-dependent amount of cash x^θ. We assume that x^G is distributed uniformly on $[\Delta x, C]$ and that $x^B = x^G - \Delta x$ where $\Delta x > 0$. Any identified excess cash is disbursed to shareholders at the end of the first period. In addition, funds can increase payouts as follows: By expending an infinitesimal non-pecuniary cost, they can make the target borrow some amount $F \in R_+$ from financiers against its second period cash flows. As a result the payout at the end of the first period is $D_1 = x^\theta + F$.[6] As noted in the

[5] While private equity funds usually obtain control as part of the LBO, activist hedge funds typically hold minority stakes but are able to wield disproportionate influence on target firms. An analysis of the latter phenomena can be found in Brav, Dasgupta, and Mathews (2017).

[6] D_1 does not literally have to be paid out to fund investors, but can instead be reinvested in other

introduction, we show that debt is the optimal contract for raising external finance in Section 4.

Long-term activism (t = 2). Suppose that activists can, in the second period, apply their skills to restructure, generate business improvements, or sell the firm. Further, the cash flows generated by such activism are affected by a state which is exogenous to the firm. There are two possible states, $s \in \{H, L\}$, with $\Pr(s = H) = \gamma_s$. The state is publicly revealed at the beginning of the second period. Following the revelation of the state, funds can exert effort $e \in \{0, \bar{e}\}$ at private cost

$$c_e = \begin{cases} 0 & \text{if } e = 0 \\ c_{\bar{e}} > 0 & \text{otherwise} \end{cases},$$

giving rise to cash flows, $X_s^\theta > 0$ with probability $\bar{e}$ and 0 otherwise. These cash flows, net of any payments to financiers, are paid out to shareholders at the end of the second period (D_2). We make standard monotonocity assumptions, i.e., $X_s^G > X_s^B$ for both s (good activists generate more cash flows than bad ones), and $X_H^\theta > X_L^\theta$ for both θ (effort generates higher cash flows in the high state).[7]

Information. Funds are the most informed party in the model. At the beginning of the first period funds learn their type θ and the realized values of x^B and x^G.[8] Investors only learn the realized values of x^B and x^G. At the end of the first period, investors see the payout D_1 and form beliefs $\mu_{IN}^{pre}(D_1) = \Pr(\theta = G|D_1)$. They may then, at private cost $c_v > 0$, verify ($a_{IN}^v = 1$) the amount of funding F (in which case they observe F perfectly,

targets on their behalf. Further, as discussed in Section 3.4, the model also allows for borrowing at the level of the fund.

[7]These payoffs imply a perfect correlation in ability (by type) across the two forms of activism. Our qualitative results only require that this correlation is sufficiently high. For example, we could allow a small probability ϵ that bad funds get lucky and generate x^G in the first period.

[8]By assuming that funds do not initially know their type we effectively rule out signalling via compensation contracts. The lack of initial self-knowledge could be understood in a broader dynamic context where new funds are born every period and incumbent funds do not know their skills relative to these newcomers.

and thus infer θ) or choose not to do so $(a^v_{IN} = 0)$. Funds have multiple methods for increasing leverage at the level of the target firm such as bank borrowing, drawing down credit lines, lengthening trade credit terms, etc. It therefore seems plausible that investors do not *costlessly* observe the precise composition of the payout in real time.[9] Following verification, the investor's beliefs are denoted by $\mu^{post}_{IN}(a^v_{IN})$ where $\mu^{post}_{IN}(0) = \mu^{pre}_{IN}(D_1)$ and $\mu^{post}_{IN}(1) \in \{0, 1\}$ since verification reveals the fund's type perfectly. They then decide whether to retain $(a^r_{IN} = 1)$ or to fire $(a^r_{IN} = 0)$ the fund. If $a^r_{IN} = 0$, the fund is shut down, and the target firm is sold to outside buyers at prices corresponding to target firm values without fund effort in the second period. Financiers do not observe the realized values of x^G, x^B, but observe F (since they are providing it). They form beliefs $\mu_{FI}(F) = \Pr(\theta = G|F)$ and set the face value K due at the end of the second period to break even, making all relevant equilibrium inferences. Financiers, like funds, investors, and target firms observe the state s at the beginning of the second period.

Fund fees. Motivated by standard compensation arrangements in the asset management industry, fees in our model are made up of two parts. The first part is an assets-under-management (AUM) fee, w, paid at the beginning of each period of employment. The second part is an incentive fee—a so-called "carry"—which is $\alpha \max(D_2, 0)$ for some $\alpha \in (0, 1)$. This implies that funds that are retained by their investors for the second period get a share of the liquidating cash flows to equity holders in addition to their second period AUM fee. The prospect of the carry and AUM fee implies that funds have an incentive to be retained by their investors and may be tempted to take actions to ensure retention. This is how we model competition for flow.

Abstracting from the first period carry is a simplification which—as will be clear later—*reduces* incentives for leveraging. Since our paper emphasizes the negative impli-

[9]Drastic examples of investors not being able to observe leverage in real time are the governance scandals of the early 2000s, e.g., Enron or Parmalat. For a related model in which the composition of financing is costlessly observed, see an early version of the paper (Burkart and Dasgupta 2015).

cations of excessive leverage induced by competition for flow, this simplification works *against* us. We also abstract from lock-up provisions. All that we require is that there is an additional payoff to a fund from being viewed as good as opposed to bad. Instead of bad funds being closed down, we could have lock-up provisions and additional inflows to those funds that are identified to be good—possibly put into a second fund run by the same manager.

Parameter restrictions. To focus on the interesting constellation of parameters, we make two assumptions. The first ensures that the free cash flow problem in the target firm is sufficiently severe by itself to make it worthwhile to engage an activist:

$$Assumption\,1 : C > \triangle x + 2w. \tag{1}$$

The second relates to restructuring:

$$Assumption\,2 : \bar{e}X_H^B < c_{\bar{e}} \leq \alpha\bar{e}X_L^G. \tag{2}$$

The inequality on the left implies that effort exertion by the bad fund is negative NPV and thus guarantees that investors would not wish to retain a bad fund if identified. If investors were to retain both good and bad funds, there is no competition for flow, eliminating the sole source of fund-level agency problems in our model. The inequality on the right excludes the possibility that the good fund does not exert effort in the low state *purely* due to the high cost of activism. Violating this inequality is tantamount to hard-wiring a connection between low states and reduced activism.

Key model ingredients. We conclude the model section by highlighting the key ingredients. A signalling model with debt overhang requires both (i) asymmetric information (for signalling) and (ii) some agency cost borne by equity holders, in our model costly effort (for debt overhang). As regards (i), funds signal to their investors by first period performance, boosting such performance as necessary by moving resources forward in time (i.e., levering up). In order for such payout boosting to have any salience for signalling (i-a) investors cannot freely observe the composition of payouts (otherwise it would be pointless to boost payout) and (i-b) financiers, who must observe the amount of debt raised cannot know exactly how much the bad type needs to borrow to imitate the good type (otherwise they could recognize and not lend to the bad type, removing the need for good types to signal).

1.1 Preliminary Analysis

A perfect Bayesian equilibrium is given by $(F^*, e^*(\cdot), a_{IN}^{v^*}, a_{IN}^{r^*}, K^*, \mu_{IN}^{pre^*}, \mu_{IN}^{post^*}, \mu_{FI}^*)$ where (i) the verification decision $a_{IN}^{v^*}$ is optimal given beliefs $\mu_{IN}^{pre^*}$, and the retention decision $a_{IN}^{r^*}$ is optimal given beliefs $\mu_{IN}^{post^*}$; (ii) The face value K^* allows the financier to break even; (iii) Funding F^* and state-contingent effort $e^*(\cdot)$ are best responses of the fund to $(a_{IN}^{v^*}, a_{IN}^{r^*}, \mu_{IN}^{pre^*}, \mu_{IN}^{post^*})$ and (K^*, μ_{FI}^*); and (iv) The beliefs $\mu_{IN}^{pre^*}, \mu_{IN}^{post^*}, \mu_{FI}^*$ are consistent with Bayes updating along the equilibrium path and are arbitrarily chosen otherwise. We begin by ruling out pooling equilibria.

Lemma 1. *For c_v sufficiently small there is no pooling equilibrium.*

All proofs are in the appendix. The intuition is as follows. In a potential pooling outcome, both types of funds must provide the same payout D_1, which in turn means that (at least) the bad fund must borrow.[10] Upon observing a pooling payout, investors must choose whether to verify or not. Whether the investor prefers to verify or to adopt an unconditional retention or firing strategy depends on the level of the verification cost. If verification is very costly, investors would retain or fire in an uncontingent manner. If verification is inexpensive, then verifying is better than uncontingent retention, because it avoids paying the fee w to bad funds in the second period. But, if the investor verifies, then there is no point for the bad fund to borrow to mimic the good fund. Even if verification costs are low, the investor may still prefer uncontingent firing to verification.

[10]Since our reasoning is based on pooling *outcomes*, it is immaterial whether funds play mixed or pure strategies. The only difference across the two cases is the probability with which a pooling outcome occurs

However, in this case again it is futile for the bad fund to borrow. Hence, the bad fund never wants to borrow at a cost as part of any strategy which leads to a pooling outcome with positive probability. Thus, pooling equilibria are excluded.

For the remainder of the paper, we assume small verification costs, which are perhaps most relevant for publicly traded firms. Under this assumption, the only possibilities are separating equilibria without verification. For brevity, we shall henceforth refer to these as separating equilibria. In what follows, we do not allow for the unrealistic possibility that all financiers commit to provide arbitrary but identical amounts of funding to each and every target firm. Therefore, we only consider equilibria without such commitment.[11]

We continue our preliminary analysis by making a few observations about separating equilibria. The corresponding results are formally stated and proved in the appendix. Since investors never knowingly retain bad funds such funds are always closed down at the end of the first period in any separating equilibrium. This means that in any separating equilibrium, the bad fund will not borrow (Claim 1). Now, since the bad fund does not borrow in a separating equilibrium, the financier will rationally assume that any positive amount F is raised by a good type (Claim 2) and therefore is willing to lend up to the (equilibrium) expected cash flows generated by the good type in the second period, which we henceforth refer to as the pledgable income of the good type (PI^G).

These observations sharply restrict the set of separating equilbria that can arise. Since the financier does not know x^B and x^G he cannot infer how much the good type would need to raise in equilibrium. Thus, the financier cannot detect potential deviations by the bad type which involve borrowing any amount up to PI^G. But this means that, to separate, the good fund must pay out an amount so high that, even by borrowing the

[11]Equilibria with commitment can formally be ruled out, for example, by imposing the requirement that financiers' beliefs are always $\mu^*_{FI}(\widehat{F}) = 1$ for all $\widehat{F} \neq F^*$. Such beliefs are compatible with the equilibria we derive below.

maximum amount possible, the bad type cannot imitate.

Lemma 2. *In separating equilibria,* $D_1^*(G) \geq x^B + PI^G$.

Except in the uninteresting case in which future cash flows that can be generated by the activist fund are so low that $x^B + PI^G \leq x^G$, i.e., that $PI^G < \Delta x$, separation requires the use of external finance. Thus, the good fund must raise external finance $F^*(G) = D_1^*(G) - x^G \geq PI^G - \Delta x$.

2 Activism, Competition, and Economic Prospects

Our preliminary analysis shows that competition for investor flow implies that good funds always separate in equilibrium, and that such separation implies borrowing. Here, in our main results, we explore the consequences of borrowing to separate.

Before stating our formal results, we introduce some suggestive terminology. To motivate this terminology, note that since the fund receives only the second-period carry, she does not wish to borrow too much: The more she borrows, the less is this carry (by definition). So, it is reasonable to focus on the separating equilibrium that delivers separation with as little leverage as possible. In addition, since—as will be clear from our result below—borrowing to separate may (under certain conditions) shut down fund activism in low states, focussing on separating equilibria with *minimal* leverage establishes the conditions under which such reduced activism is an *essential* element of equilibrium. In the remainder of the paper, we shall refer to the equilibrium which delivers separation with as little leverage as possible as the *separating equilibrium with minimal leverage* (SEML). It follows from Lemma 2, that in a SEML the good fund borrows $F^*(G) = PI^G - \Delta x$.

Proposition 1. *As long as* $\gamma_s \bar{e}\left(X_H^G - X_L^G\right) > \Delta x > \frac{w}{1-\alpha}$*, the separating equilibrium with minimal leverage exists and involves:*

(i) For $c_{\bar{e}} \in \left(0, \frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+\right)$*,* $e^*(s) = \bar{e}$ *for all* s*.*

(ii) For $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$*,* $e^*(H) = \bar{e}$ *and* $e^*(L) = 0$*.*

When effort costs are relatively low, the fund exerts effort in both states, but when effort costs are relatively high it does so only in the high state. This reduction of activist effort is, however, *not* due to high effort cost *alone*: Given Assumption (2), if the good fund were the sole claimant to the incremental cash flows generated by effort in the low state, she *would* exert effort in that state. She does not do so because, in equilibrium, she can*not* claim a sufficient fraction of the incremental cash flow due to leverage taken on to separate from the bad type. Thus, leverage induced by competition for flow generates debt overhang in the low state and shuts down activist effort.[12] Since this arises in the separating equilibrium with minimal leverage, for the relevant range of effort cost, such a state-contingent reduction of activist effort is an *essential* part of equilibrium.

The proof of this result is detailed in the appendix and heuristically summarized here. The incentive compatibility condition implies that the minimum face value which triggers debt overhang in the low state is $\underline{K} = X_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}$. Similarly, the maximum face value which ensures effort exertion in the high state is $\bar{K} = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}$. In the SEML the good fund pays out just enough to separate even if the bad fund were to borrow the full pledgeable income of the good. Hence, the good fund must use the contract with the higher pledgeable income. Otherwise, the bad type could mimic the good type's SEML payout, contradicting separation. The choice between the contract that promises $\underline{K}$ and one that promises $\bar{K}$ involves the following trade-off. On the one hand, the former contract pays less conditional on success than the latter. On the other hand, creditors are paid in full more often under the former contract (with probability $\bar{e}$) than under

[12]The reduction of activist effort due to debt overhang would arise even if effort choices were continuous. With continous effort choices, optimal effort may be higher in the high state even without leverage. Nonetheless, leverage would endogenously amplify the wedge between the effort choices.

the latter (with probability $\gamma_s \bar{e}$). This can be shown to jointly imply that the pledgeable income associated with the former contract is higher when the effort cost is low. In that case, separation involves the use of a lower face-value contract which maintains incentives to exert effort in both states. In contrast, when effort costs are relatively high, separation involves the use of a higher face-value contract which destroys incentives to exert effort in the low state. This is the dichotomy captured in the result above.

The upper and lower bounds on Δx in the condition in Proposition 1 can be understood as follows. Consider the upper bound. Lemma 2 implies that good funds must borrow $PI^G - \Delta x$ to separate. Thus, in the SEML, good funds borrow exactly $PI^G - \Delta x$, and therefore leverage is decreasing in Δx. If Δx is too large, there would be insufficient borrowing to generate debt overhang in the low state. At the same time, Δx cannot be too small, because otherwise investors would not wish to retain good funds: in the SEML all but Δx of the pledgeable income is paid out to the investor in the first period, hence retaining the good fund is only attractive if investor's second-period after-fee payoff is positive.

Last but not least, the key mechanism driving all our results is that funds compete for investor flow: it is the need to convince investors of their high ability, and thus avoid losing investment mandates, that leads good funds to lever up the target firm, generating debt overhang in the low state. If, for whatever reason, investors did not make retention decisions contingent on first-period performance, then there would be no need to lever up in order to inflate such performance, obviating our core result.

We now discuss the implications of Proposition 1 for activist effort at $t = 2$—henceforth referred to as activism for short—over the economic cycle. To do so, we henceforth interpret the state $s \in \{H, L\}$ to represent aggregate economic conditions. Given this interpretation, Proposition 1 implies that fund activism is procyclical: it always occurs when economic conditions are good but if conditions are poor, activism ceases unless costs are low.

Proposition 1 also identifies the range of activism costs, $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$, which make activism *fragile*, in the sense that the fund only exerts effort in the high state. For future reference we label the cost range over which activism is fragile as Condition **FR**:

Definition 1. Condition **FR** holds if and only if $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$.

While Proposition 1 delineates the relationship between *realized* economic conditions and activism, we now turn to the role of economic *prospects* as captured by $\gamma_s = Pr(s = H)$.[13] Our second result traces the role of economic prospects in affecting the fragility of activism, its profitability, and target firm leverage.

Proposition 2. *Aggregate economic prospects affect activism as follows:*
(a) Better economic prospects ensure that ***FR*** *holds for a wider range of costs, making activism more fragile.*
(b) When ***FR*** *holds,*
  (i) Returns from investing in activist funds are increasing in economic prospects.
  (ii) Target firm leverage is increasing in economic prospects.

Economic prospects affect the lower bound of **FR**, $\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+$, at which activist effort becomes fragile. Upon inspection of this condition, it is clear that this threshold is 0 for $\gamma_s \geq {}^{X_L^G}/_{X_H^G}$ (so that activism is procyclical for all possible cost levels), attains a maximum of $\alpha\bar{e}X_L^G$ for $\gamma_s = 0$ (so that activism is never procyclical), and is decreasing in γ_s for intermediate values. Proposition 2(a) thus implies that economic prospects exacerbate the fragility of activism. Thus, when economic prospects seem

[13]For simplicity, we interpret state s as a purely aggregate state in this section. In general for a single firm, the probability of attaining the high cash flow in the second period (γ_s) depends both on the aggregate economic state and on idiosyncratic firm-level events. However, even so, an improvement in the aggregate state would affect the prospects of a large number of firms, and can be observed as procyclical activism in the data.

particularly good—say, during a bull (equity) market—even relatively low cost forms of interventions may not be immune to an economic downturn ex post.

Proposition 2(b.i) states that when **FR** holds, returns from investing in activist funds are increasing in economic prospects. At an intuitive level, two elements drive this result. First, conditional on being matched with a good fund, the investor receives a payout in the first period that is positively linked to the firm's pledgable income under the stewardship of the good fund. Since the good fund produces cash flows only in the good state under **FR**, this pledgable income is increasing in the probability of the good state arising. Second, upon reaching the second period, matched to a good fund, the investor—as an equity holder—receives a positive cash flow only if the firm does not default, which again arises only in the good state under **FR**.

Proposition 2(b.ii) states that when **FR** holds, target firms will be more highly levered. Intuitively, better economic prospects imply a higher debt capacity for the target, which in turn implies that more borrowing is necessary for good type funds to separate. Consistent with Proposition 2(b.ii), Axelson et al. (2013) report that private equity buyout leverage is procyclical. It also helps us interpret some anecdotal evidence with respect to the post-crisis behavior of activist hedge funds around 2010. *The Economist* writes at the time: "Activists are toning down their attempts to get companies to take on more debt. Many were burned before, and are reluctant to put their hands back in the fire."[14] Our model suggests that this may simply be a case of lower market confidence about future prospects for the economy in 2010 than in the heady days of optimism prior to the financial crisis.

3 Activist Hedge Funds

Activism by hedge funds represents a good illustration of our theory. On the one hand, the mitigation of free cash flow problems is a central goal of these funds. As Brav,

[14] *The Economist*, "Shareholder activism: Ready, set dough", December 2, 2010.

Jiang, and Kim (2010) note in their survey, hedge fund targets can be characterised as "... "cash-cows" with low growth potentials that may suffer from the agency problem of free cash flow." On the other hand, longer-term forms of activism by hedge funds often include changes in business strategy and the sale of target companies. Such changes, taken together, constitute almost half of 13D filings.[15]

In this section, we delve deeper into hedge fund activism. As a first step, in section 3.1 we relate our model predictions to available empirical and anecdotal evidence on activist hedge funds. In section 3.2 we provide a minor variation of our model to examine how hedge fund activism affects target firm *bond*holders. Third, in section 3.3 we study a related model variation in which our core results obtain through changes in payout policy alone while holding leverage constant. Hence, we can interpret our results more generally in terms of *net* debt. Finally, in section 3.4, we argue that our model can be more broadly interpreted.

3.1 Interpreting the empirical evidence

Two key applied themes emerge from our analysis. First, since activist funds enhance payouts via increased net leverage, target firms experience increases in payout and leverage. Second, as a result of the procyclicality discussed above, investment in activist funds are higher in bull markets. Both implications resonate with the available empirical evidence on activist hedge funds.

The empirical literature suggests that activist hedge funds increase target firm leverage or payout or both (e.g. Brav et al. 2008; Klein and Zur 2009). There is also evidence—consistent with our results—that the induced rise in leverage increases the

[15]Our model assumes that a fund potentially engages in more than one form of activism. This is consistent with Brav, Jiang, and Kim (2010). In their sample 52% of 13D filings declare specific goals falling into four categories but the percentages of 13D filings, when summed over specific goals, amount to nearly 85%. Thus, *on average*, hedge funds state close to two distinct activist goals per 13D declaration.

credit risk of target firms: Target companies disproportionately experience credit downgrades (e.g., Byrd, Hambly, and Watson 2007; Klein and Zur 2011). Our model, of course, also suggests that the increase in leverage induced by activists potentially undermines future value creation at the level of target firms. This view receives support from prominent market participants. For instance, Larry Fink, the chairman of BlackRock, wrote to executives of all portfolio firms in the context of hedge fund activism that "Too many companies have... increased debt to boost dividends", and that such actions "can jeopardize a company's ability to generate sustainable long-term returns."[16]

There is also growing evidence that activist investments are higher in bull markets. See, for example, Figures 1 and 2 in Brav, Jiang, and Kim (2013) which depict the number of activist hedge funds and their engagement disclosures (e.g., 13D filings) over time in the US. These findings are echoed in the financial press. According to *The Economist*, "In America investors began only two new activist campaigns in the fourth quarter of 2008, down from 32 in the preceding nine months and 61 in 2007."[17] It is only after a "strangely quiet" period during the two years following this steep decline in activism, during which "[m]any [activist investors] scaled back or even closed shop,"[18] that activist campaigns started to re-emerge. Indeed, it is only another eighteen months later, in mid-2012, when the market had regained most of the value lost in the 2008 crisis, that – according to Peter Harkins of D.F. King, a proxy-advisor – shareholder activism is "getting back to normal after the financial crisis of 2008."[19] Further supporting evidence from more recent years can be found in Khorana, Shivdasani, and Sigurdsson (2017).

It is sometimes suggested in the financial press that the procyclicality of returns from activist hedge funds is caused by the relative lack of diversification of activist portfolios.[20] Further, since one of the commonly declared objectives of activist hedge

[16] *The Wall Street Journal*, 21 March 2014.

[17] *The Economist*, "Activist Investors: Flight of the Locusts", April 8, 2009.

[18] *The Economist*, "Shareholder activism: Ready, set dough", December 2, 2010.

[19] *The Economist*, "Corporate Governance in America: Heating Up," April 7, 2012.

[20] It is worth noting that an explanation based upon idiosyncratic shocks is hard to square with

funds is the eventual sale of the target firm, it may also be tempting to attribute the procyclicality of hedge fund activism to the procyclicality of M&A markets. While these other potential channels may have a bearing on the procyclicality of activism, it is worth emphasizing that our analysis—apart from delivering a self-contained model with fully rational agents—delivers an endogenous link between the observed procyclicality of activism and the documented effect of activism on the net debt of target firms.

Our model can also shed light on the broader debate on whether hedge fund activism creates value or not. A number of legal scholars and commentators (see Kahan and Rock 2007 for an overview) argued that hedge fund activists are short-termist and destroy firm value. The financial economics literature establishes comprehensively that activism is value enhancing both in the short-term (e.g. Brav et al. 2008) and in the long term (e.g. Bebchuk, Brav, and Jiang 2015). Based on this latter evidence we make modeling choices that ensure that activism is overall beneficial to target firm value (please see Proposition 7 in the appendix for details). In particular, activism leads to the resolution of the free cash flow problem resulting in a positive net cash flow in the first period, which is further enhanced by a leveraged payout if the fund present is of the good type. Furthermore, the good fund generates a positive second period net cash flow in the high state, and also in the low state if effort costs are small. However, our model also highlights that activism comes with endogenously generated costs. Due to competition for flow, activists are short-termist—they overlever target firms to pay their investors early. Further, activism amplifies the exposure of firm level variables to aggregate economic states, fostering fragility. Thus, activism is not a panacea for addressing managerial agency problems.

3.2 Do activists expropriate bondholders?

There is general agreement in the literature that—as in our model—hedge fund activism produces significant positive returns to target shareholders. However, the empirical

patterns related to the business cycle.

literature is not unanimous on whether (some of) these gains are at the expense of existing bondholders. On the one hand, Klein and Zur (2011) argue that hedge fund activism leads to an expropriation of existing bondholders. On the other, Brav et al. (2008) argue that expropriation of existing bondholders is unlikely to be a source of significant shareholder value because they find that returns to target shareholders are *higher* in companies which are previously *un*levered.

Our core mechanism does not turn on the interaction between existing bondholders and shareholders: Since the representative target firm is unlevered in our model, our baseline results are silent on the issue of bondholder expropriation. Nevertheless, our framework can be used to interpret the seemingly conflicting evidence in Brav et al. (2008) and Klein and Zur (2011). Reconsider the baseline model with the following modifications. Assume that the representative firm has some liquid assets of $Y_0 > 0$ in the first period. Unlike the pre-existing excess cash C, which is subject to a free cash flow problem, these liquid assets Y_0 are not under the target firm manager's discretion and thus cannot be wasted. Thus, absent hedge fund activists, this Y_0 would be retained until the second period and available to pay pre-existing creditor claims, if any. Hedge fund activists may pay out part or all of these liquid assets in the first period to enhance early returns to their investors, in addition to leveraging the target as in the baseline model. As before, investors do not directly verify the composition of the payout but infer it in equilibrium. We compare two capital structures for the target firm: Either the target firm has no pre-existing debt (as in the baseline model) or it has pre-existing debt maturing in the second period with a face value of $K_0 \in (\Delta x, Y_0)$. We slightly modify Assumption 2 to account for pre-existing debt and liquid assets Y_0 and assume $\bar{e}\left(X_H^B - K_0 + Y_0\right) < c_{\bar{e}} \leq \alpha\bar{e}\left(X_L^G - K_0 + Y_0\right)$.

Proposition 3. *For* $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$ *and* $\gamma_s\bar{e}\left(X_H^G - X_L^G\right) > \Delta x > \frac{w}{1-\alpha}$*, pre-existing target leverage may reduce shareholder returns from activism even when activism expropriates existing bondholders.*

Using arguments that parallel those of Proposition 1, we show in the appendix that competition for flow induces the good fund to pay out all available liquid assets in the first period and also to leverage the target sufficiently to generate debt overhang in the low state in the second period. This implies that activist funds reduce the cash available for existing creditors: In the absence of hedge funds, pre-existing debt is safe and creditors are paid in both states. In the presence of hedge funds, the pre-existing debt becomes risky and creditors are only paid with probability $\bar{e}$ in the high state, consistent with the findings of Klein and Zur (2011). However, comparing target firms with and without pre-existing leverage in the presence of activist funds, Proposition 3 shows that returns to shareholders are higher when the target firm is unlevered. This is because pre-existing target debt reduces the (residual) debt capacity of the target, which in turn reduces the payout necessary for separation and hence the equilibrium first period payout to target firm shareholders. The second period payout is unaffected because activist funds borrow all but Δx of the target's debt capacity. Hence, in the presence of activist funds, returns are lower to the target firm shareholders when there is pre-existing leverage, consistent with the findings of Brav et al. (2008). Thus, our model provides a simple, stylized framework that helps to resolve some of the seemingly contradictory empirical evidence in Brav et al. (2008) and Klein and Zur (2011).

3.3 Excessive payout

The enriched framework introduced in section 3.2 delivers a further benefit: It enables us to show that our results hold if we restrict hedge funds to changing payout policy *only*, i.e., preclude them from issuing *new* target debt. Consequently, our results can be interpreted more broadly in terms of increases in *net* debt – i.e., debt minus cash – thus linking them more directly to evidence on increased payout (Brav et al. 2008; Klein and Zur 2009).

Our results are indeed robust to payout policy changes provided that target firms have both pre-existing debt and liquid assets: For targets with pre-existing debt, a reduction in liquid assets increases net debt. Competition for flow can deliver sufficiently high net debt to foster debt overhang in the low state. We consider the same variation of the model as in section 3.2 except that new borrowing is precluded. Activist hedge funds salvage excess cash of x^θ and pay it out at the end of the first period. They may augment the payment by tapping into liquid assets Y_0. In the absence of a hedge fund activist, the liquid assets Y_0 would be retained until the second period and available to pay pre-existing creditor claims.

Proposition 4. *Competition for flow leads to high payouts which in turn may cause debt overhang even without new target firm borrowing.*

The intuition is that—as before—good funds must pay high enough dividends at the end of the first period to prevent mimicking by bad funds. Since either fund can tap into the liquid assets, the good fund must pay out at least $x^B + Y_0$ to separate, i.e., can retain only $\triangle x$ liquid assets. But, then, for target firms with sufficient pre-existing leverage, debt overhang arises in the low state.

3.4 Broader Model Interpretations

In our model there are two periods and aggregate economic variation arises only in the second one. Needless to say, one can interpret the state of the economy in the second period as being *relative* to its state in the first. We can then view our current first period analysis as being conditional on a realised first-period state. Given any such state in the first period, the economy may improve or decline in the second. This means that, in principle, returns from both first- and second-period activism could be made state dependent without altering our qualitiative results. This paves the way for a broader intepretation of our two forms of activism. This is because the remaining difference across the two forms of activism—namely, the effort required to undertake them—can also be relaxed.

Our formal analysis assumes, purely for simplicity, that there is no effort cost associated with the first form of activism, which we have interpreted as free cash flow mitigation. Nothing would change if free cash flow mitigation requires effort and funds learn their types in the first period as effort is exerted. It would still remain the case, that in equilibrium the good funds would lever up to an extent that bad funds are unable to match the enhanced dividend. Since, therefore, both forms of activism can be costly and generate state-dependent returns, neither the sequence nor the labels given to the two forms of activism are critical for the core mechanism. The assumed sequence of free cash flow mitigation and restructuring can be reversed. For example, restructuring via potential spin-offs of non-core assets could occur in the first period with costly capital structure adjustments occuring later. Activism would still be procyclical, since leverage generated in an attempt to boost restructuring returns in the first period would interfere with capital stucture adjustments in the second.

Indeed, it is not even necessary that the activist fund potentially intervenes in two different ways in the same target firm, as in the model. Consider instead a setting in which each fund has a portfolio of target firms, intervening (in one way or the other) only once per firm, in different periods for different firms. Procyclicality would still emerge in such a setting if leverage is undertaken at the fund level rather than at the target firm level. Competition for flow would still tempt funds into enhancing early returns to investors by levering up. Under qualitatively similar conditions, endogenously generated leverage would be sufficient to discourage activists funds from exerting effort in *any* portfolio firm that subsquently required costly intervention if aggregate economic conditions decline. Note that since borrowing at the hedge fund level is also not fully transparent, it is reasonable to assume that it is at least somewhat costly for investors to verify the source of returns, as in the baseline model, giving rise to endogenous opacity as before.

4 Financing and compensation contracts

In our analysis to date, we have imposed two key contracting restrictions: (i) activists use target debt to inflate early payouts and (ii) are compensated via an assets under management fee (w) and a carry (α). While both of these contracts are well justified on empirical grounds, we now show that a single contracting friction can simultaneously rationalize both of them.

The friction we introduce is that contracts can only be made contingent on project success or failure, not on aggregate economic states ($s \in \{H, L\}$). That is, while aggregate states are publicly observable at the beginning of period 2 they are not verifiable. In practice, agents can relatively easily contract on aggregate indices (e.g., S&P 500) which reflect future economic prospects.[21] However, it is difficult to contract in real time on current aggregate states due to measurement difficulties (Shiller 1998).[22] For example, GDP — a key measure of macroeconomic states — is often revised with substantial delay. As Orphanides (2001) points out with respect to real-time Taylor rules: "... as is well known, the actual variables required for implementation of such a rule — potential output, nominal output, and real output — are not known with any accuracy until much later."

Given the non-contractibility of aggregate states, we show that debt is the optimal form of financing at the level of the target firm and there is no loss of generality in restricting the compensation contract to an AUM fee with a second-period carry.

It is well known that with binary outcomes, where the failure cash flow is zero, debt and equity are indistinguishable (Tirole 2006, p. 119). Therefore, we enrich the

[21]In our model, contracts are indeed de facto contingent on economic prospects, γ_s, which – as discussed in Section 2 – is our proxy for an aggregate market index. This is because the pledgable income PI^G varies with γ_s.

[22]Shiller (1998), p. 2: "These economic causes of changes in standards of living that should be insurable without moral hazard because they are beyond individual control are still not insurable today because they are not so objective or easy to verify as fires or disabilities."

set of cash flows generated in the second period, while keeping the rest of the model unchanged. We assume that effort level e gives rise to cash flow, $\bar{X}_s^\theta$ with probability e and $\underline{X}_s^\theta > 0$ with probability $1-e$. As before, we impose standard monotonocity assumptions: $\bar{X}_s^\theta > \underline{X}_s^\theta$ for all θ, s (fund effort increases cash flow), $\bar{X}_s^G > \bar{X}_s^B$ for all s (good funds are better than bad ones), and $\bar{X}_H^\theta > \bar{X}_L^\theta$ for all θ (effort generates higher cash flows in the high state).

Since contracts can only be made contingent on project success or failure, the differences between cash flows across aggregate states in the event of either success $(\bar{X}_H^G - \bar{X}_L^G)$ or failure $(\underline{X}_H^G - \underline{X}_L^G)$ are non-verifiable and hence divertible. Following the corporate governance literature (Shleifer and Vishny 1997), we assume that divertible cash flows accrue to the controlling party, which here is the activist fund (see the discussion in Section 1).[23]

We also have to take a stand on whether the divertible component of cash flows is higher in the event of project success or failure. We assume the former:[24]

$$Assumption\,3 : \bar{X}_H^G - \bar{X}_L^G > \underline{X}_H^G - \underline{X}_L^G. \tag{3}$$

This assumption is *not* sufficient for our results, because—as we point out below—without (endogenous) leverage generated by competition for flow, activist effort and investor returns would not be procyclical. Finally, our Assumption 2 has to be adjusted to the richer payoff structure as follows:

$$Assumption\,4 : \bar{e}\left(\bar{X}_H^B - \underline{X}_H^B\right) < c_{\bar{e}} \leq \alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right). \tag{4}$$

To avoid a plethora of cases, we make a minor simplification by setting $\bar{X}_H^B = \underline{X}_H^B$. Since Assumption 4 already precludes effort by the bad fund in the high state, this

[23]Had we explicitly modeled target firm managers, such divertible cash flows could be shared between firms and funds, without qualitatively affecting the incentives of the fund.

[24]In the reverse case, the divertibility of cash flows would make failure more attractive, undermining effort provision regardless of the aggregate state.

simplification is entirely without loss of generality.

We first note that both Lemma 1 and Lemma 2 hold in our richer setting (see Appendix section 6.2 for details). This means that there are no pooling equilibria, and—in separating equilibria—the good fund must pay out an amount so high that, even by raising the maximum amount of external financing possible, the bad type cannot imitate.

We now solve for the optimal contract for external financing. At the time of investing F, financiers set the repayments $R\left(\bar{X}_s^\theta\right)$ and $R\left(\underline{X}_s^\theta\right)$ due at the end of the second period to break even, making all relevant equilibrium inferences.

Proposition 5. *Debt is the optimal contract for raising external funding F.*

Since project success/failure is verifiable but the state is not, promised repayments can take on at most two possible values, say $\bar{R}$ and $\underline{R}$. Conditional on separation (which eliminates the bad fund in the first period) the future cash flows are increasing in the good fund's effort. Thus, we look for $\bar{R}$ and $\underline{R}$ which maximize the good fund's incentives to exert effort. While effort is costly for the fund, it allows it to obtain an $\alpha-$share of a larger cash flow with probability $\bar{e}$. In addition, the fund—having de facto control—can appropriate all non-verifiable cash flows (like the borrower/entrepreneur in e.g., Bolton and Scharfstein 1990). In particular, if the project succeeds, the fund can appropriate $\bar{X}_H^G - \bar{X}_L^G$ whereas if it fails it can appropriate $\underline{X}_H^G - \underline{X}_L^G$. Since effort increases the probability of success from 0 to $\bar{e}$, in the high state effort also generates an additional payoff of $\bar{e}\left(\left(\bar{X}_H^G - \bar{X}_L^G\right) - \left(\underline{X}_H^G - \underline{X}_L^G\right)\right)$ to the fund. Thus, as the proof in the appendix shows, the incentive compatibility constraints of the good fund are:

$$\alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) \geq c_{\bar{e}} \text{ in state } s = L, \text{ and} \tag{5}$$

$$\alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) \geq c_{\bar{e}} \text{ in state } s = H. \tag{6}$$

For arbitrarily chosen parameters, these two constraints are clearly most slack if $\bar{R} - \underline{R}$ is minimized. Imposing monotonicity, as is standard in this literature following Innes (1990), leads to two possible optimal financing arrangements: If the fund raises less than $\underline{X}_L^G$, we have safe debt with repayment $\bar{R} = \underline{R} < \underline{X}_L^G$. Otherwise, optimal external financing is achieved via defaultable debt with $\bar{R} > \underline{R} = \underline{X}_L^G$.[25]

We now re-derive our core result, Proposition 1, in this richer setting, labelling it Proposition $1'$ for ease of reference. For technical reasons, the subsequent analysis needs to be split into two cases:

$$\text{Case A: } \left(\bar{X}_H^G - \underline{X}_H^G\right) \geq (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right) \tag{7}$$

and

$$\text{Case B: } \left(\bar{X}_L^G - \underline{X}_L^G\right) < \left(\bar{X}_H^G - \underline{X}_H^G\right) < (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right). \tag{8}$$

Since α is typically on the order of 0.2 for funds, Case B is restrictive. Accordingly, we only discuss Case A in the body of the paper and relegate Case B to the appendix, where we show that the economic content of our results is essentially identical across the cases.

Proposition $1'$. *As long as $\gamma_s(1-\gamma_s)\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right) > \Delta x > \frac{w}{1-\alpha}$, the separating equilibrium with minimal leverage exists and involves:*

i. For $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$, $e^*(s) = \bar{e}$ for all s.

ii. For $c_{\bar{e}} \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$, $e^*(H) = \bar{e}$ and $e^*(L) = 0$.

The full set of our applied implications stated above which follow from Proposition 1, follow similarly in this richer setting from Proposition $1'$. We do not therefore restate these implications. Instead, we conclude this section by considering whether debt overhang in the low state could be avoided by using a more sophisticated compensation contract for the fund.

[25] Needless to say, absent the contracting friction that underlies all results in this section, state contingent debt would be the optimal contract which, by virtue of being state contingent, would rule out debt overhang.

Proposition 6. *There is no fund compensation contract that simultaneously*

i. Prevents mimicking by bad funds, and

ii. Generates effort by good funds in both states for the full range of effort costs that—absent leverage—would induce effort in the low state.

Given the non-verifiable component of returns in the high state, the bad fund always has an incentive to try to survive into the second period: Irrespective of the second period contractual payments, survival enables the bad fund to effortlessly earn at least an expected payoff of $\gamma_s \left(\underline{X}_H^B - \underline{X}_L^B\right) > 0$. As a result, she tries to mimic the good fund. In turn, the good fund levers up to separate. Whenever she levers, there is *some* cost range for which she subsequently does not work in the low state while she would have done so unlevered. Thus, the non-contractibility of economic states implies that there is no contract between investors and funds that can preclude debt overhang in the low state.

5 Conclusions

We propose a model of activism by asset managers in the presence of competition for flows. Our self-contained theory highlights how agency frictions arising out of the delegation of portfolio management can affect the nature of blockholder monitoring and, more broadly, may help to enrich our understanding of corporate governance issues. In addition, our model suggests a new channel by which the incentives of asset managers can amplify booms and busts and foster economic fragility. In addition to these broader implications, our paper sheds light on the observed procyclicality of hedge fund activism and reconciles it with the documented effect of activist hedge funds on the net leverage of their target firms. Finally, we generate some testable implications which resolve some contradictory empirical evidence on the wealth effects of hedge fund activism on different stakeholders in target firms.

6 Appendix

Proof of Lemma 1: In pooling outcomes we must have $D_1^G = D_1^B := D_1^P$ and $F(B) > 0$. Since we condition our argument only on pooling outcomes, we do not need to impose any assumption about whether funds randomize over borrowing amounts. In case one or both funds randomize, the pooling outcome can only occur with some probability.

Let the gross (of verification cost) expected payoff to the investor if he verifies be Π_v. Following verification, investors can retain or fire the fund in a type dependent manner, and therefore their payoff will be:

$$\Pi_v = \gamma_\theta max\left(\Pi^G\left(D_1\right), 0\right) + \left(1 - \gamma_\theta\right)(0),$$

where $\Pi^G\left(D_1\right)$ denotes the investor's expected second period net payoff from retaining a good fund given D_1 and 0 is the price at which the firm can be sold. Since the bad type never makes an effort in the second period, she is always fired and the firm is sold. The good type may or may not be retained, depending on whether $\Pi^G\left(D_1\right)$ is larger or smaller than 0. Without verification if the investor always retains, we denote the expected payoff by Π_1 or if he always fires, we denote the expected payoff by Π_0. We have:

$$\Pi_1 = \gamma_\theta \Pi^G\left(D_1\right) + \left(1 - \gamma_\theta\right)(-w),$$

and

$$\Pi_0 = 0.$$

It is clear that $\Pi_v \geq \Pi_0$ and $\Pi_v > \Pi_1$. Suppose that c_v is sufficiently small to satisfy $c_v < \Pi_v - \Pi_1$. Then there are two possibilities. Either, $\Pi_v - c_v > max(\Pi_0, \Pi_1)$ and the investor would verify if $D_1^G = D_1^B := D_1^P$, so there is no point to borrowing and thus $F\left(B\right) = 0$ and $D_1(G) \neq D_1(B)$, contradicting pooling. Or, we have $\Pi_0 > \Pi_v - c_v > \Pi_1$ and both types are fired without verification. Consequently, again $F(G) = F(B) = 0$ and $D_1(G) \neq D_1(B)$, contradicting pooling.

If investors randomised between retaining and firing without verification then it must be the case $\Pi_0 = \Pi_1$. However, then we have that $\Pi_v - c_v > \Pi_0 = \Pi_1$ and the investor would indeed verify if $D_1^G = D_1^B := D_1^P$. Finally, we consider the possibility of the investors randomizing between verification and uncontingent retention or firing. Clearly, for $c_v < \Pi_v - \Pi_1$, investors will never randomize between verification and uncontingent retention. If the investor were to randomize between verification and firing without verification, in neither case does the bad type wish to borrow, and hence again $D_1(G) \neq D_1(B)$.■

Claim 1. If $D_1^*(G) \neq D_1^*(B)$, then $F^*(B) = 0$.

Proof of Claim 1: If $D_1^*(G) \neq D_1^*(B)$, then $\mu_{IN}^{pre^*}(D_1^*(B)) = 0$. Assumption (2) implies that bad funds does not exert effort. Thus, investors fire the bad fund, because by doing so they save the fee, w, to be paid in the second period. Thus, $a_{IN}^{r^*}(D_1^*(B)) = 0$, and $F^*(B) = 0$ since choosing $F > 0$ creates an infinitesimal cost for the fund.■

Claim 2. If $D_1^*(G) \neq D_1^*(B)$, then $\mu_{FI}^*(F) = 1$ for $F \in \left(0, PI^G\right]$.

Proof of Claim 2: The equilibrium payout $D_1^*(G)$ can be represented as a map $f : \left(x^G, x^B\right) \to \mathbb{R}_+$. The required borrowing is therefore $F^*(G) = f\left(x^G, x^B\right) - x^G$. Except in the special case in which $f\left(x^G, x^B\right) - x^G = k$ for some $k \in \mathbb{R}$ – which by definition can only arise in equilibria in which financiers commit/coordinate to lend only specific amounts and are thus ruled out in our analysis – financiers cannot compute $F^*(G)$ before the funding request is made because they do not know x^G . However, since $F^*(B) = 0$ (Lemma 1), any requested amount $F \in \left(0, PI^G\right]$ is consistent with $\mu_{FI}^*(F) = 1$.■

Proof of Lemma 2: Since in a separating equilibrium $\mu_{FI}^*(F) = 1$ for $F \in \left(0, PI^G\right]$, financiers are willing to lend up to PI^G. Suppose that $D_1^*(G) < x_1^B + PI^G$. Then, type B can deviate and raise $D_1^*(G) - x^B < PI^G$ and successfully imitate type G violating $D_1^*(G) \neq D_1^*(B)$.■

Proof of Proposition 1: The derivation proceeds in three steps.

Step 1: Debt Overhang thresholds

For a given face value of debt K debt overhang arises in state $s = L$ only if

$$\alpha\bar{e}\left(X_L^G - K\right) < c_{\bar{e}}.$$

Thus, the maximum face value of debt associated with effort exertion in state $s = L$ is

$$\underline{K} = X_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}.$$

Conversely, for a given face value K, there is no debt overhang in state $s = H$ if

$$\alpha\bar{e}\left(X_L^G - K\right) \geq c_{\bar{e}}.$$

Thus, the maximum face value of debt associated with effort exertion in state $s = H$ is

$$\bar{K} = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}.$$

Step 2: Pledgeable Income PI^G

We compare the maximum pledgable income with debt $\underline{K}$ and the one with debt $\bar{K}$. Without debt overhang in state $s = L$ pledgeable income is equal to $PI^G_{\underline{K}} = \bar{e}\underline{K}$. With debt overhang in state $s = L$ pledgable income is equal to $PI^G_{\bar{K}} = \gamma_s\bar{e}\bar{K}$. Then $PI^G_{\bar{K}} > PI^G_{\underline{K}}$ is equivalent to

$$c_{\bar{e}} \geq \frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+.$$

Step 3(a): Equilibrium borrowing and retention given that $PI^G_{\bar{K}} > PI^G_{\underline{K}}$

Separation requires borrowing of

$$PI^G_{\bar{K}} - \Delta x = \gamma_s\bar{e}\bar{K} - \Delta x,$$

and the corresponding face value K^* is:

$$K^* = \frac{\gamma_s\bar{e}\bar{K} - \Delta x}{\gamma_s\bar{e}} = \frac{\gamma_s\bar{e}\left(X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) - \Delta x}{\gamma_s\bar{e}} = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x}{\gamma_s\bar{e}}.$$

For consistency we need $K^* > \underline{K}$, i.e.,

$$\frac{\gamma_s \bar{e} \bar{K} - \Delta x}{\gamma_s \bar{e}} > \underline{K},$$

i.e.,

$$\gamma_s \bar{e} \left(X_H^G - X_L^G \right) \geq \Delta x.$$

It remains to check that it is in the investor's interest to retain a good fund. Retention results in a payoff equal to

$$(1 - \alpha) \gamma_s \bar{e} \left(X_H^G - K^* \right) - w,$$

Liquidating the fund/firm results in a payoff of 0. Retention requires:

$$(1 - \alpha) \gamma_s \bar{e} \left(X_H^G - K^* \right) \geq w$$

which, upon substituting in the value of K^* is equivalent to:

$$(1 - \alpha) \gamma_s \bar{e} \left(X_H^G - X_H^G + \frac{c_{\bar{e}}}{\alpha \bar{e}} + \frac{\Delta x}{\gamma_s \bar{e}} \right) \geq w.$$

Now, using the lower bound (for debt overhang) of effort costs yields:

$$\Delta x \geq \frac{w}{1 - \alpha} - \frac{\gamma_s \bar{e}}{1 - \gamma_s} \left(X_L^G - \gamma_s X_H^G \right)^+.$$

Step 3(b): Equilibrium borrowing and retention given that $PI_{\bar{K}}^G < PI_{\underline{K}}^G$

Proposition 2 implies that separation requires borrowing of

$$PI_{\underline{K}}^G - \Delta x = \bar{e} \left(X_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) - \Delta x,$$

and the corresponding face value is

$$K^{**} = \frac{\bar{e} \left(X_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) - \Delta x}{\bar{e}} = X_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} - \frac{\Delta x}{\bar{e}}.$$

It remains to check that it is in the investor's interest to retain a good fund. Retaining the good fund generates a continuation payoff equal to

$$(1 - \alpha) \bar{e} \left(X_L^G - K^{**} + \gamma_s \left(X_H^G - X_L^G \right) \right) - w,$$

which must be compared to a payoff of 0 for firing. Simplifying, retention requires that:

$$\Delta x \geq \frac{w}{1-\alpha} - \frac{c_{\bar{e}}}{\alpha} - \bar{e}\gamma_s \left(X_H^G - X_L^G\right).$$

This concludes the proof of the proposition.■

Proof of Proposition 2:

Part (a). This follows from the fact that $\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+$ is (i) decreasing in γ_s for $\gamma_s \in [0, X_L^G/X_H^G)$, (ii) 0 for $\gamma_s \geq X_L^G/X_H^G$, and (iii) $\alpha\bar{e}X_L^G$ for $\gamma_s = 0$.

Part (b.i). For $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$, the investor's expected cash flows are:

$$\gamma_\theta E\left(x^G\right) + (1-\gamma_\theta) E\left(x^B\right) - w + \gamma_\theta\left(PI^G - \Delta x + (1-\alpha)\gamma_s\bar{e}\left(X_H^G - K^*\right) - w\right),$$

i.e., the sum of net payoffs from free cash flow mitigation and restructuring. Using from the proof of Proposition 1 the facts that

$$PI_{\bar{K}}^G - \Delta x = \gamma_s\bar{e}\bar{K} - \Delta x,$$

$$\bar{K} = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}},$$

and

$$K^* = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x}{\gamma_s\bar{e}},$$

the investor's expected cash flows can be simplied to:

$$\gamma_\theta E\left(x^G\right) + (1-\gamma_\theta) E\left(x^B\right) - w + \gamma_\theta\left(\gamma_s\left(\bar{e}X_H^G - c_{\bar{e}}\right) - \alpha\Delta x - w\right), \tag{9}$$

which is clearly increasing in γ_s since $\bar{e}X_H^G - c_{\bar{e}} > 0$ by (2).

Part (b.ii). The amount of borrowing in the SEML is $PI_{\bar{K}}^G - \Delta x = \gamma_s\bar{e}\left(X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) - \Delta x$, while the face value of the debt is $K^* = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x}{\gamma_s\bar{e}}$. Both quantities are increasing in γ_s. In addition, end of the first period ratio of the market value of debt to the market value of the firm is $\frac{PI_{\bar{K}}^G - \Delta x}{PI_{\bar{K}}^G} = 1 - \frac{\Delta x}{PI_{\bar{K}}^G}$ is also increasing in γ_s.■

Proof of Proposition 3: To separate, the good fund must pay out enough to prevent mimicking by the bad fund. The good fund always prefers to pay out liquid assets Y_0 in the first period (that would anyway go to creditors in the second period) because, holding fixed the separation payout, replacing the paying out of Y_0 with additional borrowing is costly: For each dollar borrowed the good fund must pay back either $1/\gamma_s \bar{e}$ (if debt overhang arises) or $1/\bar{e}$ (otherwise) in the second period. Both are costly to the hedge fund's payoff, as it receives a second period carry. This establishes that Y_0 is fully paid out in any separating equilibrium. The remaining steps mirror those of the proof of Proposition 1, and are thus stated in brief.

Given pre-existing debt K_0 and all liquid assets Y_0 paid out, there is debt overhang in $s = L$ if the face value of debt satisfies $K > \underline{K}_{K_0} \equiv X_L^G - K_0 - \frac{c_{\bar{e}}}{\alpha \bar{e}}$, and no debt overhang in $s = H$ if $K < \bar{K}_{K_0} = X_H^G - K_0 - \frac{c_{\bar{e}}}{\alpha \bar{e}}$. For

$$c_{\bar{e}} \in \left[\frac{\alpha \bar{e}}{1 - \gamma_s} \left(X_L^G - \gamma_s X_H^G - (1 - \gamma_s) K_0 \right)^+ , \alpha \bar{e} \left(X_L^G - K_0 + Y_0 \right) \right]$$

it is easy to check that $PI^G_{\bar{K}_{K_0}} \geq PI^G_{\underline{K}_{K_0}}$. Thus, separation requires an amount of borrowing equal to $PI^G_{\bar{K}_{K_0}} - \Delta x = \gamma_s \bar{e} \left(X_H^G - K_0 - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) - \Delta x$, with corresponding face value $K^*_{K_0} = X_H^G - K_0 - \frac{c_{\bar{e}}}{\alpha \bar{e}} - \frac{\Delta x}{\gamma_s \bar{e}}$. For consistency we need $K^*_{K_0} > \underline{K}_{K_0}$, which is always satisfied as long as $\gamma_s \bar{e} \left(X_H^G - X_L^G \right) > \Delta x$, as in the baseline model.

Next we check that the investor wants to retain a good hedge fund. Since w paid at $t = 1$ is sunk and the investor has already received $D_1^* = x^G + Y_0 + \gamma_s \bar{e} \left(X_H^G - K_0 - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) - \Delta x$, the investor retains the good fund if $(1 - \alpha) \gamma_s \bar{e} \left(X_H^G - K_0 - K^*_{K_0} \right) \geq w$, which is guaranteed if i.e., if $\Delta x > \frac{w}{1 - \alpha}$ as in the baseline model.

For $c_{\bar{e}} \in \left[\frac{\alpha \bar{e}}{1 - \gamma_s} \left(X_L^G - \gamma_s X_H^G \right)^+ , \alpha \bar{e} X_L^G \right]$, the analysis of the baseline model implies that debt overhang arises in the low state in the SEML in the unleveraged target firm (since Y_0 is paid out in the first period). Further, since $\frac{\alpha \bar{e}}{1 - \gamma_s} \left(X_L^G - \gamma_s X_H^G \right)^+ \geq \frac{\alpha \bar{e}}{1 - \gamma_s} \left(X_L^G - \gamma_s X_H^G - (1 - \gamma_s) K_0 \right)^+$ and $\alpha \bar{e} X_L^G < \alpha \bar{e} \left(X_L^G - K_0 + Y_0 \right)$, we can conclude that for $c_{\bar{e}} \in \left[\frac{\alpha \bar{e}}{1 - \gamma_s} \left(X_L^G - \gamma_s X_H^G \right)^+ , \alpha \bar{e} X_L^G \right]$, debt overhang arises in the low state in the SEML in levered and unlevered target firms.

Finally, we can compare (i) the payoffs to equity holders in firms with and without pre-existing debt in the presence of hedge fund activists and (ii) the payoffs to pre-existing creditors in levered target firms in the presence and absence of hedge fund activists.

(i) **Payoffs to equity holders:** With pre-existing leverage of K_0, target shareholders receive an expected payoff of

$$\gamma_\theta \left(E\left(x^G\right) + Y_0 + \gamma_s \bar{e} \left(X_H^G - K_0 - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) - \Delta x \right) + (1 - \gamma_\theta) E\left(x_1^B\right)$$

in the first period and $\gamma_\theta \left(\gamma_s \frac{c_{\bar{e}}}{\alpha} + \Delta x \right)$ in the second period. Without leverage, target shareholders receive an expected payoff of

$$\gamma_\theta \left(E\left(x^G\right) + Y_0 + \gamma_s \bar{e} \left(X_H^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) - \Delta x \right) + (1 - \gamma_\theta) E\left(x_1^B\right)$$

in the first period and $\gamma_\theta \left(\gamma_s \frac{c_{\bar{e}}}{\alpha} + \Delta x \right)$ in the second period. Thus, pre-existing leverage reduces first period payoffs to target shareholders without affecting second period payoffs.

(ii) **Payoffs to pre-existing creditors:** In the absence of the hedge fund activists, creditors would have expected to receive K_0 in the second period in either state (since $Y_0 > K_0$). In the presence of hedge fund activists, the same creditors can expect to receive K_0 in the second period in the high state with probability $\bar{e}$ but nothing otherwise. Thus, the presence of activist hedge funds expropriates pre-existing creditors.■

Proof of Proposition 4: To separate, the good type has to pay out $D_1^*(G) = x^B + Y_0$ and can therefore retain at most $x^G + Y_0 - \left(x^B + Y_0\right) = \Delta x$ liquid assets. For

$$K_0 \in \left(X_L^G + \triangle x - \frac{c_{\bar{e}}}{\alpha \bar{e}}, X_H^G + \triangle x - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right)$$

the incentive compatibility constraint in state $s = L$

$$\alpha \bar{e} (X_L^G - K_0 + \Delta x) \geq c_{\bar{e}}$$

is violated but that for state $s = H$

$$\alpha \bar{e} (X_H^G - K_0 + \Delta x) \geq c_{\bar{e}}$$

is satisfied.■

6.1 Activism and target firm value

In our model, activism is hampered by the incentives of funds to compete for flow. Nevertheless, target firms are better off with activist funds than without.

Proposition 7. *Activism increases total cash flows generated by the target firm.*

Proof: From the proof of Proposition 2(b.i), the investor's expected cash flows under condition **FR** is

$$\gamma_\theta E\left(x^G\right) + \left(1-\gamma_\theta\right) E\left(x^B\right) - w + \gamma_\theta\left(\gamma_s\left(\bar{e}X_H^G - c_{\bar{e}}\right) - \alpha\Delta x - w\right).$$

The former is positive because $E\left(x^G\right) > E\left(x^B\right) = \frac{C-\triangle x}{2} > w$ by Assumption 1. Inserting the highest effort cost, the latter can be further rearranged as follows:

$$\gamma_s\alpha\bar{e}\left(X_H^G - X_L^G - \Delta x\right) + \left(1-\alpha\right)\left(\bar{e}X_H^G - \frac{w}{1-\alpha}\right),$$

where both terms in the parentheses are positive under the conditions of Proposition 1. The investor's expected cash flow in equilibrium is clearly higher in the case of smaller effort costs.■

6.2 Proofs for Section 4

The statement of Lemma 1 is unchanged. The proof requires the following modifications. Now, the payoff following verification is:

$$\Pi_v = \gamma_\theta max\left(\Pi^G\left(D_1\right), P^G\right) + \left(1-\gamma_\theta\right)P^B,$$

where $\Pi^G\left(D_1\right)$ denotes the investor's expected second period net payoff from retaining a good fund given D_1 and $P^\theta = max\left(\underline{X}_L^\theta - R\left(\underline{X}_L^\theta\right), 0\right)$ is the type-dependent price at which the target can be sold. (The earlier proof of Lemma 1 shows that this proof holds for a type-uncontingent sales price as well.) Since the bad type never makes an effort in the second period, she is always fired and the firm is sold. The good type may or may not be retained, depending on whether the investors receive enough from the additional cash flows generated by the good type's effort. Without verification the investor may always retain (with expected payoff Π_1) or always fire (with expected payoff Π_0). We have:

$$\Pi_1 = \gamma_\theta \Pi^G (D_1) + (1-\gamma_\theta) \left(max \left(\underline{X}_L^B - R \left(\underline{X}_L^B \right), 0 \right) (1-\alpha) - w \right),$$

and

$$\Pi_0 = \gamma_\theta P^G + (1-\gamma_\theta) P^B.$$

Since $max \left(\Pi^G (D_1), P^G \right) \geq \Pi^G (D_1)$ and

$$P^B = max \left(\underline{X}_L^B - R \left(\underline{X}_L^B \right), 0 \right) > max \left(\underline{X}_L^B - R \left(\underline{X}_L^B \right), 0 \right) (1-\alpha) - w,$$

it follows again that $\Pi_1 < \Pi_v$. Since verification does not preclude the option of always firing both types, we have again that $\Pi_v \geq \Pi_0$. Given these rankings, the end of the proof of Lemma 1 applies.■

The statements and proof of Lemma 2 remain unchanged.

Proof of Proposition 5: Since there are four possible cash flows generated by the good type (two states crossed with project success or failure), the repayment function $R(\cdot)$ takes four possible values: $R\left(\bar{X}_L^G\right)$, $R\left(\bar{X}_H^G\right)$, $R\left(\underline{X}_L^G\right)$, and $R\left(\underline{X}_H^G\right)$ respectively. The verifiability of project success coupled with the non-verifiability of realized cash flows implies that

$$R\left(\bar{X}_L^G\right) = R\left(\bar{X}_H^G\right) := \bar{R} \text{ and } R\left(\underline{X}_L^G\right) = R\left(\underline{X}_H^G\right) := \underline{R}.$$

It also implies that in state H the fund captures the incremental cash flows $\bar{X}_H^G - \bar{X}_L^G$ and $\underline{X}_H^G - \underline{X}_L^G$ conditional on success and failure respectively, since investors cannot verify whether $s = H$ or L.

Effort exertion in state $s = L$ requires that

$$\begin{aligned} \alpha \left(\bar{e} \left(\bar{X}_L^G - \bar{R} \right) + (1-\bar{e}) \left(\underline{X}_L^G - \underline{R} \right) \right) - c_{\bar{e}} &\geq \alpha \left(\underline{X}_L^G - \underline{R} \right), \\ \text{i.e., } \alpha \bar{e} \left(\left(\bar{X}_L^G - \underline{X}_L^G \right) - \left(\bar{R} - \underline{R} \right) \right) &\geq c_{\bar{e}}. \end{aligned} \quad (10)$$

Effort exertion in state $s = H$ requires that

$$\begin{pmatrix} \alpha\bar{e}\left(\bar{X}_L^G - \bar{R}\right) + \bar{e}\left(\bar{X}_H^G - \bar{X}_L^G\right) + \\ \alpha\left(1-\bar{e}\right)\left(\underline{X}_L^G - \underline{R}\right) + \left(1-\bar{e}\right)\left(\underline{X}_H^G - \underline{X}_L^G\right) \end{pmatrix} - c_{\bar{e}} \geq \alpha\left(\underline{X}_L^G - \underline{R}\right) + \left(\underline{X}_H^G - \underline{X}_L^G\right),$$

$$\text{i.e., } \alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) \geq c_{\bar{e}}. \quad (11)$$

For arbitrarily chosen parameters, (10) and (11) are clearly most slack if $\bar{R} - \underline{R}$ is minimized. Imposing monotonicity implies $\bar{R} \geq \underline{R}$. Hence, if the fund raises less than $\underline{X}_L^G$, we have safe debt with repayment $\bar{R} = \underline{R} < \underline{X}_L^G$. Otherwise, optimal external financing is achieved via defaultable debt with $\bar{R} > \underline{R} = \underline{X}_L^G$, i.e., the face value of debt must be $K \geq \underline{X}_L^G$. The maximum (fulfillable) face value of debt is given by $K \leq \bar{X}_L^G$.■

Proof of Proposition 1′:

Step 1: Debt Overhang thresholds

For a given face value of debt K debt overhang arises in state $s = L$ only if

$$\alpha\left[\bar{e}\left(\bar{X}_L^G - K\right) - \bar{e}\left(\underline{X}_L^G - \min(K, \underline{X}_L^G)\right)\right] < c_{\bar{e}}.$$

For $K < \underline{X}_L^G$ the above reduces to $\alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right) \leq c_{\bar{e}}$, which violates Assumption (4). Thus, $K > \underline{X}_L^G$, and the maximum face value of debt associated with effort exertion in state $s = L$ is

$$\underline{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}.$$

For a given face value K, there is no debt overhang in state $s = H$ if

$$\begin{pmatrix} \alpha\left[\bar{e}\left(\bar{X}_L^G - K\right) + \left(1-\bar{e}\right)\left(\underline{X}_L^G - \min(\underline{X}_L^G, K)\right)\right] \\ +\bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) \end{pmatrix} - c_{\bar{e}} \geq \alpha\left(\underline{X}_L^G - \min(\underline{X}_L^G, K)\right)$$

Since we look for debt levels that induce debt overhang in state $s = L$, $K > \underline{K} > \underline{X}_L^G$ so that the expression above simplifies to:

$$\alpha\bar{e}\left(\bar{X}_L^G - K\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) - c_{\bar{e}} \geq 0,$$

which gives us

$$K \le \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right).$$

If

$$c_{\bar{e}} \le \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$$

then the relevant constraint for K is

$$K \le \bar{X}_L^G,$$

because of the non-verifiability of economic states. Assumption (4) guarantees that

$$c_{\bar{e}} \le \alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right).$$

Thus, if

$$\begin{aligned} \alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right) &< \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right), \\ \text{i.e., } \left(\bar{X}_H^G - \underline{X}_H^G\right) &\ge (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right), \end{aligned}$$

then, under Assumption (2) the relevant constraint for K is always

$$K \le \bar{X}_L^G.$$

and

$$\bar{K} = \bar{X}_L^G.$$

Step 2: Pledgeable Income PI^G

To derive the conditions under which pledgable income is higher, we compare the maximum pledgable income with debt $\underline{K}$ and the one with debt $\bar{K}$. Without debt overhang in state $s = L$ pledgeable income is equal to

$$\bar{e}\underline{K} + (1-\bar{e})\,\underline{X}_L^G.$$

Inserting $\underline{K} = \bar{X}_L^G - c_{\bar{e}}/\alpha\bar{e}$ yields the maximum pledgeable income $PI_{\underline{K}}^G$:

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G.$$

With debt overhang in state $s = L$ pledgable income is equal to

$$\gamma_s \bar{e} \bar{K} + (1 - \gamma_s \bar{e}) \underline{X}_L^G.$$

Inserting the expression for $\bar{K} = \bar{X}_L^G$ yields the maximum pledgeable income $PI_{\bar{K}}^G$:

$$PI_{\bar{K}}^G = \gamma_s \bar{e} \bar{X}_L^G + (1 - \gamma_s \bar{e}) \underline{X}_L^G.$$

Then $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ is equivalent to

$$c_{\bar{e}} \geq (1 - \gamma_s) \alpha \bar{e} \left(\bar{X}_L^G - \underline{X}_L^G \right).$$

Thus, for $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ the maximum pledgeable income is $PI_{\underline{K}}^G$ (Case A.1), while for $c_{\bar{e}} \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$, the maximum pledgeable income is $PI_{\bar{K}}^G$ (Case A.2).

Case A.1: $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$

Step 3 for A.1: Funding amount for $PI_{\bar{K}}^G < PI_{\underline{K}}^G$

Lemma 2 implies that separation requires borrowing of

$$PI_{\underline{K}}^G - \Delta x = \bar{e} \left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) + (1 - \bar{e}) \underline{X}_L^G - \Delta x,$$

and the corresponding face value K^{**} solves

$$\bar{e} \left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) + (1 - \bar{e}) \underline{X}_L^G - \Delta x = \bar{e} K^{**} + (1 - \bar{e}) \min(K^{**}, \underline{X}_L^G). \tag{12}$$

Suppose $K^{**} > \underline{X}_L^G$, then $\min(K^{**}, \underline{X}_L^G) = \underline{X}_L^G$, in which case (12) gives:

$$K^{**} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} - \frac{\Delta x}{\bar{e}},$$

which is clearly smaller than $\underline{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}$ so that there is indeed no debt overhang in state $s = L$. Furthermore, the condition $\bar{X}_L^G > \underline{X}_L^G + \frac{\Delta x}{\gamma_s(1-\gamma_s)\bar{e}}$ ensures that $K^{**} > \underline{X}_L^G$.

Indeed, a sufficient condition for $K^{**} > \underline{X}_L^G$ for all $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ is that

$$\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x}{\bar{e}} > \underline{X}_L^G$$

for $c_{\bar{e}} = (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]$. This in turn, is equivalent to:

$$\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x}{\gamma_s \bar{e}} \tag{13}$$

which always holds since $\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x}{\gamma_s(1-\gamma_s)\bar{e}} > \frac{\Delta x}{\gamma_s \bar{e}}$.

It remains to check that it is in the investor's interest to retain a good fund. Retaining the good fund generates a continuation payoff equal to

$$(1-\alpha)\,\bar{e}\left(\bar{X}_L^G - K^{**}\right) - w,$$

which does not depend on the aggregate state due to a combination of (i) no debt overhang and (ii) non verifiability of the state. Liquidating the fund/firm results in a payoff of $\max\left(\underline{X}_L^G - K^{**}, 0\right) = 0$. Thus retention requires:

$$(1-\alpha)\left(\frac{c_{\bar{e}}}{\alpha} + \Delta x\right) - w \geq 0 \tag{14}$$

which is clearly always satisfied given $\Delta x > \frac{w}{1-\alpha}$. This concludes the proof of the proposition for constellation A.1.

Case A.2: $c_{\bar{e}} \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$

Step 3 for A.2: Funding amount given that $PI_{\bar{K}}^G > PI_{\underline{K}}^G$

Separation requires borrowing of

$$PI_{\bar{K}}^G - \Delta x = \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G - \Delta x,$$

and the corresponding face value K^* is obtained by setting

$$\gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G - \Delta x = \gamma_s\bar{e}K^* + (1-\gamma_s\bar{e})\,\underline{X}_L^G,$$

giving

$$K^* = \frac{\gamma_s \bar{e} \bar{X}_L^G - \Delta x}{\gamma_s \bar{e}} = \bar{X}_L^G - \frac{\Delta x}{\gamma_s \bar{e}}.$$

For consistency we need $K^* > \underline{K}$, i.e.,

$$\bar{X}_L^G - \frac{\Delta x}{\gamma_s \bar{e}} > \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}},$$

i.e.,

$$\Delta x < \frac{\gamma_s}{\alpha} c_{\bar{e}}$$

Since $c_{\bar{e}} \geq (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]$, the constraint above is always satisfied given

$$\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x}{\gamma_s(1-\gamma_s)\bar{e}}. \tag{15}$$

It remains to check that it is in the investor's interest to retain a good fund. Retaining the fund results in a payoff equal of

$$(1-\alpha)\left(\gamma_s\left(\bar{e}\left(\bar{X}_L^G - K^*\right) + (1-\bar{e})\max\left(\underline{X}_L^G - K^*, 0\right)\right) + (1-\gamma_s)\max\left(\underline{X}_L^G - K^*, 0\right)\right) - w,$$

Liquidating the fund/firm results in a payoff of

$$\max\left(\underline{X}_L^G - K^*, 0\right).$$

Since $K^* = \bar{X}_L^G - \frac{\Delta x}{\gamma_s \bar{e}} > \underline{K} > \underline{X}_L^G$, the investor retains the good fund if:

$$(1-\alpha)\gamma_s\bar{e}\left(\bar{X}_L^G - \bar{X}_L^G + \frac{\Delta x}{\gamma_s \bar{e}}\right) - w \geq 0 \tag{16}$$

which is clearly satisfied given $\Delta x > \frac{w}{(1-\alpha)}$. This concludes the proof of the proposition for case A.2.■

Case B: $\left(\bar{\mathbf{X}}_{\mathbf{L}}^{\mathbf{G}} - \underline{\mathbf{X}}_{\mathbf{L}}^{\mathbf{G}}\right) < \left(\bar{\mathbf{X}}_{\mathbf{H}}^{\mathbf{G}} - \underline{\mathbf{X}}_{\mathbf{H}}^{\mathbf{G}}\right) < (\mathbf{1}+\alpha)\left(\bar{\mathbf{X}}_{\mathbf{L}}^{\mathbf{G}} - \underline{\mathbf{X}}_{\mathbf{L}}^{\mathbf{G}}\right)$

When $\left(\bar{X}_L^G - \underline{X}_L^G\right) < \left(\bar{X}_H^G - \underline{X}_H^G\right) < (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right)$, there are two possibilities: For $c_{\bar{e}} \leq \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$, $\bar{K} = \bar{X}_L^G$, while for $c_{\bar{e}} > \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$, $\bar{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$.

For $c_{\bar{e}} \leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, $\bar{K} = \bar{X}_L^G$ while $\underline{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}$ as before. Consequently,

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$$

and

$$PI_{\bar{K}}^G = \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G$$

As in case **A1)**, the condition for $PI_{\bar{K}}^G \geq PI_{\underline{K}}^G$ is

$$c_{\bar{e}} \geq (1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G]$$

Since $c_{\bar{e}} \leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, this condition can only be satisfied if

$$\begin{aligned}(1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G] &\leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\\ \gamma_s &\geq 1 - \frac{1}{\alpha}\left[\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} - 1\right] := \tilde{\gamma}_s.\end{aligned}$$

Note that $\tilde{\gamma}_s \to 0$ as $\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} \to 1+\alpha$ and $\tilde{\gamma}_s \to 1$ as $\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} \to 1$ so $\gamma_s \in [0,1]$. Thus, for $\gamma_s < \tilde{\gamma}_s$ the maximum pledgeable income is $PI_{\underline{K}}^G$ for all $c_{\bar{e}} \in \left(0, \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$. For $\gamma_s \geq \tilde{\gamma}_s$, the maximum pledgeable income is $PI_{\underline{K}}^G$ for $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G]\right)$ and $PI_{\bar{K}}^G$ for $c_{\bar{e}} \in \left((1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G], \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$. To ensure debt overhang in the latter case, the face value associated with raising $F = PI_{\bar{K}}^G - \Delta x$ has to be larger than $\underline{K}$. As shown in case A.2 (step 4) above, this holds for $\Delta x < \frac{\gamma_s}{\alpha}c_{\bar{e}}$ which is again guaranteed by (15).

For $c_{\bar{e}} \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$, $\underline{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}$ as before and $\bar{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$. Consequently,

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$$

and

$$PI_{\bar{K}}^G = \gamma_s \bar{e} \left[\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} + \frac{1}{\alpha} \left(\left(\bar{X}_H^G - \underline{X}_H^G \right) - \left(\bar{X}_L^G - \underline{X}_L^G \right) \right) \right] + (1 - \gamma_s \bar{e}) \underline{X}_L^G$$

Hence, $PI_{\bar{K}}^G \geq PI_{\underline{K}}^G$ holds if

$$\gamma_s \bar{e} \left[\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} + \frac{1}{\alpha} \begin{pmatrix} (\bar{X}_H^G - \underline{X}_H^G) \\ -(\bar{X}_L^G - \underline{X}_L^G) \end{pmatrix} \right] + (1 - \gamma_s \bar{e}) \underline{X}_L^G \geq \bar{e} \left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) + (1 - \bar{e}) \underline{X}_L^G$$

i.e.,

$$\gamma_s \geq \frac{\bar{X}_L^G - \underline{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}}}{\frac{1}{\alpha} \left(\left(\bar{X}_H^G - \underline{X}_H^G \right) - \left(\bar{X}_L^G - \underline{X}_L^G \right) \right) + \left(\bar{X}_L^G - \underline{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right)} := \hat{\gamma}_s \in (0, 1) \, .$$

Thus, in the range $c_{\bar{e}} \in \left(\bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right], \alpha \bar{e} (\bar{X}_L^G - \underline{X}_L^G) \right)$ the maximum pledgeable income is $PI_{\underline{K}}^G$ for $\gamma_s < \hat{\gamma}_s$ and $PI_{\bar{K}}^G$ for $\gamma_s \geq \hat{\gamma}_s$. To ensure debt overhang in the latter case, the face value associated with raising $F = PI_{\bar{K}}^G - \Delta x$ has to be larger than $\underline{K}$. As shown in case A.2 (step 4) above, this holds for $\Delta x < \frac{\gamma_s}{\alpha} c_{\bar{e}}$ which is again guaranteed by (15).

We now establish that $\tilde{\gamma}_s \geq \hat{\gamma}_s$. Suppose the reverse were true, i.e., $\tilde{\gamma}_s < \hat{\gamma}_s$ and consider $\gamma_s \in (\tilde{\gamma}_s, \hat{\gamma}_s)$ and effort costs immediately to the left and right of the threshold $\bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right]$. Since $\gamma_s > \tilde{\gamma}_s$, for $c_{\bar{e}} = \bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right] - \epsilon$ for some small $\epsilon > 0$, $PI_{\bar{K}}^G > PI_{\underline{K}}^G$. Yet, since $\gamma_s < \hat{\gamma}_s$, for $c_{\bar{e}} = \bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right] + \epsilon$, $PI_{\bar{K}}^G < PI_{\underline{K}}^G$. Note that $PI_{\underline{K}}^G$ is given by $\bar{e} \left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} \right) + (1 - \bar{e}) \underline{X}_L^G$ for all $c_{\bar{e}}$ and decreases in $c_{\bar{e}}$ at the rate $1/\alpha$.

In contrast, for $c_{\bar{e}} \in \left[\bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right] - \epsilon, \bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right] \right)$, $PI_{\bar{K}}^G$ is given by $\gamma_s \bar{e} \bar{X}_L^G + (1 - \gamma_s \bar{e}) \underline{X}_L^G$ which is invariant with $c_{\bar{e}}$. For $c_{\bar{e}} \in \left(\bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right], \bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right] + \epsilon \right]$, $PI_{\bar{K}}^G$ is given by $\gamma_s \bar{e} \left[\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} + \frac{1}{\alpha} \left(\left(\bar{X}_H^G - \underline{X}_H^G \right) - \left(\bar{X}_L^G - \underline{X}_L^G \right) \right) \right] + (1 - \gamma_s \bar{e}) \underline{X}_L^G$ which decreases in $c_{\bar{e}}$ at the rate γ_s / α, i.e., more slowly than $PI_{\underline{K}}^G$ in the same interval. Thus if $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ for $c_{\bar{e}} = \bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right] - \epsilon$ it must also be true that $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ for $c_{\bar{e}} = \bar{e} \left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G) \right] + \epsilon$, a contradiction.

To summarize our findings, we have three regions in terms of γ_s:

1. If $\gamma_s < \hat{\gamma}_s$, then $PI^G_{\bar{K}} < PI^G_{\underline{K}}$ for the full relevant range of $c_{\bar{e}}$ and there is no debt overhang.

2. If $\hat{\gamma}_s \leq \gamma_s < \tilde{\gamma}_s$, then for $c_{\bar{e}} \in \left(0, \bar{e}\left[(\bar{X}^G_H - \underline{X}^G_H) - (\bar{X}^G_L - \underline{X}^G_L)\right]\right)$ we have $PI^G_{\bar{K}} < PI^G_{\underline{K}}$ and no debt overhang, while for $c_{\bar{e}} \in \left(\bar{e}\left[(\bar{X}^G_H - \underline{X}^G_H) - (\bar{X}^G_L - \underline{X}^G_L)\right], \alpha\bar{e}(\bar{X}^G_L - \underline{X}^G_L)\right)$ we have $PI^G_{\bar{K}} > PI^G_{\underline{K}}$ and debt overhang.

3. If $\tilde{\gamma}_s \leq \gamma_s$, then for $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}[\bar{X}^G_L - \underline{X}^G_L]\right)$ we have $PI^G_{\bar{K}} < PI^G_{\underline{K}}$ and no debt overhang, while for $c_{\bar{e}} \in \left((1-\gamma_s)\alpha\bar{e}[\bar{X}^G_L - \underline{X}^G_L], \alpha\bar{e}(\bar{X}^G_L - \underline{X}^G_L)\right)$ we have $PI^G_{\bar{K}} > PI^G_{\underline{K}}$ and debt overhang.

It remains to check that it is in the investor's interest to retain a good fund. In all three regions of γ_s where $PI^G_{\underline{K}} > PI^G_{\bar{K}}$ the analysis of the retention decision is identical to case A.1 (step 4). In the regions $\gamma_s < \hat{\gamma}_s$ and $\gamma_s \geq \tilde{\gamma}_s$ where $PI^G_{\bar{K}} > PI^G_{\underline{K}}$ the constraint $\bar{K} = \bar{X}^G_L$ binds, and the analysis of the retention decision is identical to case A.2. (step 4). In the region $\gamma_s \in [\hat{\gamma}_s, \tilde{\gamma}_s)$ where $PI^G_{\bar{K}} > PI^G_{\underline{K}}$ the constraint $\bar{K} = \bar{X}^G_L - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}^G_H - \underline{X}^G_H\right) - \left(\bar{X}^G_L - \underline{X}^G_L\right)\right)$ binds. The corresponding face value of debt K^{***} is obtained by setting

$$\begin{array}{c} \gamma_s\bar{e}\left[\bar{X}^G_L - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}^G_H - \underline{X}^G_H\right) - \left(\bar{X}^G_L - \underline{X}^G_L\right)\right)\right] \\ + \left(1 - \gamma_s\bar{e}\right)\underline{X}^G_L - \Delta x \end{array} = \gamma_s\bar{e}K^{***} + \left(1 - \gamma_s\bar{e}\right)\underline{X}^G_L,$$

giving

$$K^{***} = \bar{X}^G_L - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}^G_H - \underline{X}^G_H\right) - \left(\bar{X}^G_L - \underline{X}^G_L\right)\right) - \frac{\Delta x}{\gamma_s\bar{e}}.$$

Hence, the investor's payoff from retaining the fund is

$$(1-\alpha)\left[\bar{X}^G_L - \left(\bar{X}^G_L - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}^G_H - \underline{X}^G_H\right) - \left(\bar{X}^G_L - \underline{X}^G_L\right)\right) - \frac{\Delta x}{\gamma_s\bar{e}}\right)\right] - w$$

and retention is in the investor's interest if

$$\left[\frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{1}{\alpha}\left(\left(\bar{X}^G_H - \underline{X}^G_H\right) - \left(\bar{X}^G_L - \underline{X}^G_L\right)\right) + \frac{\Delta x}{\gamma_s\bar{e}}\right] \geq \frac{w}{(1-\alpha)}$$

Since $c_{\bar{e}} > \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, this condition is satisfied given $\Delta x > \frac{w}{(1-\alpha)}$. This concludes the analysis for case B.■

Proof of Proposition 6: Consider any arbitrary contract with non-negative payments $w_1, \alpha_1(D_1), w_2(D_1), \alpha_2(D_1, X)$ where $X \in \left\{\underline{X}_L^B, \bar{X}_L^B, \underline{X}_L^G, \bar{X}_L^G\right\}$. First note that, for any $w_2(D_1), \alpha_2(D_1, X)$ the investor fires the bad fund if identified. Since the bad fund does not exert effort at $t = 2$, this is immediate for $max\left(w_2(D_1), \alpha_2(D_1, X)\right) > 0$. Even if $max\left(w_2(D_1), \alpha_2(D_1, X)\right) = 0$, the investor is strictly indifferent, and so (by our tie-breaking assumption) fires. Since the investor fires if the bad fund is identified, it is easy to see that the bad fund wishes to mimick for any $w_2(D_1), \alpha_2(D_1, X)$. For $max\left(w_2(D_1), \alpha_2(D_1, X)\right) > 0$, this is immediate, but even if $max\left(w_2(D_1), \alpha_2(D_1, X)\right) = 0$, survival enables the bad fund to effortlessly earn at least an expected payoff of $\gamma_s\left(\underline{X}_H^B - \underline{X}_L^B\right) > 0$. Finally, given that the bad fund has the incentive to mimic, the good fund has to lever to separate. Whenever she levers, there is *some* cost range for which she does not work in the low state with leverage when she would have without.■

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