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Practical Algebra and Hydrostatics: the Legacy of Thomas Harriot

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This article provides a glimpse of the unpublished work of Harriot's colleague Walter Warner. The main topic is the use of hydrostatic weighing and algebraic symbolism to determine the constitution of a mixture of metals. This was an important matter in the context of contemporary debates about currency and exchange. Recent discoveries have revealed the extent of Warner's writings in this field, and the involvement of other notable seventeenth century figures, such as Charles Thynne, Henry Robinson, John Dury, Samuel Hartlib, Justinian Isham, and David Ramsay. Their work illustrates many facets of scientific enquiry at that time, such as the contemporary concepts of weight and density, and the introduction of the new symbolic algebra in Harriot's style.

Keywords: hydrostatics, weighing, algebra, Walter Warner

1 Harriot on weighing

Thomas Harriot (1560-1621) studied many scientific problems, several of which required the accurate determination of weight. For example, his work on refraction, which led him to discover 'Snell's Law', involved accurate measurements of density. On a more mundane topic, he made very precise measurements of the weight of gold coins.[1]

In Harriot's day, weighing was viewed in the context of pre-Newtonian mechanics. The weight of an object was considered to be an inherent property, not the result of a force acting upon it, but rather a measurement obtained with a specific device. For scientific purposes the usual device was an equal-arm balance, equipped with a set of weights constructed in conformity with a fixed standard, such as the troy pound. This was the basis of the medieval science of weights, founded in antiquity and transmitted via the Islamic world to Western Europe.[2] Around 1600 another method was being developed, based on the Principle of Archimedes.. By weighing an object immersed in water, as well as in air, its density could be determined and the material of which the object was made could be found (see Section 3 for more details). This technique was employed by Harriot in several of his experiments, including his investigations of the gold coinage. He was one of the pioneers of this *hydrostatic* method, but he was not the only one, and his notes contain several references to contemporary sources.[3]

Harriot received little credit for his mathematical and scientific discoveries, because none of them were published in his lifetime. In 1610 Sir William Lower bewailed the fact that many of his discoveries had been published by others. He wrote

> So you taught me the curious way to observe weight in Water, and within a while after Ghetaldi comes out with it, in print. A little before Vieta prevented you of the Gharland for the greate invention of Algebra.[4]

It is worth noting that Lower regarded the hydrostatic method as one of Harriot's major achievements, alongside his "greate invention" of an efficient system of symbolic algebra.

After Harriot's death in 1621 attempts were made to publish some of his work. Eventually, in 1631, a book based on his research in symbolic algebra appeared. It was prepared for publication by Walter Warner, who had known Harriot for many years, and who had a broad knowledge of scientific matters. Unfortunately the book did not do justice to Harriot's achievements. Jacqueline Stedall has written that Warner

> not only destroyed the coherence of Harriot's treatise, but made it appear less sophisticated than in fact it actually was.[5]

Subsequently, Warner was involved in efforts to make more of Harriot's work available, but nothing was published. Indeed, all Harriot's papers disappeared completely until 1784, when an enormous accumulation of them was discovered at Petworth House in Sussex. And it was not until 2011 that it became possible to appreciate the full significance of his work, thanks to the online archive (http://echo.mpiwg-berlin.mpg.de).

2 Warner's *Praxis Statica*

Although Warner continued to work on Harriot's papers until his death in 1643, the extent of his labour has only gradually become clear. About 25 years ago Timothy Raylor noticed that some papers in Warner's hand were preserved in the Isham Collection at the Northamptonshire Record Office (NRO). and recent discoveries have shed light on the provenance of this material. In order to illustrate the importance of the NRO archive we shall give here an exposition of a small part of it.[6] The topic is the coinage of precious metals, a matter that was constantly debated in the first half of the 17th century because of its significance for the wealth of the nation.

Item IL3422 at the NRO is a set of Notebooks, mainly in Warner's hand. Notebook 13 contains material that derives from two of Harriot's major innovations, hydrostatic weighing and his system of symbolic algebra. The main part of it is in Latin, but there is some particularly interesting material in English at the end. The Latin section has a cover page bearing the words *Praxis Statica*, and the first page of text is headed *Ad praxim Staticam Elementa quaedam accomoda.* The title is reminiscent of the *Artis analyticae praxis ad aequationes algebraicas nova*, the exposition of Harriot's work on symbolic algebra produced by Warner in 1631. Thus it is possible that the *Praxis Statica* is part of an attempt by Warner to produce a similar exposition of Harriot's work on hydrostatics.

The *Praxis Statica* is written in the style of a medieval tract, similar to the thirteenth-century manuscripts on weighing ascribed to Jordanus of Nemore.[7] There are three definitions, followed by problems, propositions, conclusions, and a set of diagrams. The definitions are apparently intended to serve as axioms, and it is not entirely clear what is being defined and what is being assumed. There are two theorems. *Theorema prima* is a statement of Archimedes Principle (unsurprisingly, no proof is offered). *Theorema secunda* does not involve the hydrostatic method, but it is followed by a proof containing a significant application of algebra. The topic is the weighing of mixtures.

Warner considers an object that is known to be a mixture of two substances. The weight P and the volume C of this object are known. If L and R are the weights of the constituent substances, what is the ratio L : R ? The method is to assume that two separate objects made of the constituents are available, both having the same weight P as the given object, and volumes B and D. Warner denotes the volumes of the constituents in the mixture by F and G, and writes down the basic equations

$$L + R = P, \quad F + G = C, \quad L : R = P : B, \quad R : G = P : D.$$

After some algebra, with many deletions and corrections, he obtains the correct result

$$L : R = C\text{-}D : B\text{-}C.$$

The *Praxis Statica* is undated, but there is a scribal copy of it in the British Library from which we can infer that it was probably written before 1635.[8] The BL copy is one of a group of three items, the last of which is inscribed "Per D. Gualterum Warner Feb. 12, 1634". Two of these items are stated to be transcriptions by Huntington Smithson, who was employed by the Cavendish family on their estates in Nottinghamshire. Indeed, there are letters from Sir Charles Cavendish and his associate Robert Payne to Warner that refer to some of these tracts.[9] In one of these letters, Payne observes

> the theorems you take for principles, undemonstrated, require demonstration as much as the conclusion you would prove from them.[10]

It seems likely that the Harleian copies were passed down through generations of the Cavendish family and eventually came into the possession of Margaret Cavendish Holles Harley, the daughter of Edward Harley and Henrietta Cavendish Holles, who arranged for the transfer of the Harley collection of manuscripts to the British Museum in 1753.[11] It has been suggested that the material on weighing silver would have been useful to the Cavendishes in the Civil War, when Charles's brother William was head of the royalist armies in the North, and had to mint coins to pay the troops.[12]

The Harleian copy of the *Praxis Statica* and the original at the NRO are both unfinished. However, the NRO version has some additional material in English, which suggests that Warner remained hopeful about publishing a more complete exposition of Harriot's work on this subject.

3 Additions to the *Praxis Statica*

The core of the present article is an analysis of two folios of English text that are attached to the *Praxis Statica*, as it is now preserved at the NRO. These pages overlap with parts of the *Praxis,* but they are not a verbatim translation of it. The first folio begins with a clear statement of Archimedes Principle (Figure 1).

> Yf a solid body of hevier substance then water be severally waighed in the aire and also in the water, the excesse of the waight thereof in the air above the waight thereof in water is the waight of so much water as is equall in magnitude to the solid body so waighed.

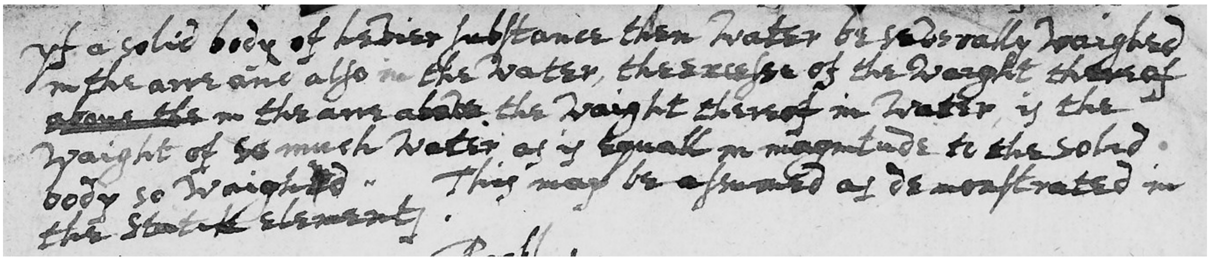

Yf a solid body of hevier substance then water be severally waighed in the aire and also in the water, the excesse of the waight thereof above the in the aire above the waight thereof in water is the waight of so much water as is equall in magnitude to the solid body so waighed. This may be assumed as demonstrated in the Statik elements.

Figure 1. Warner’s statement of Archimedes Principle. NRO IL3422, Notebook 13, f.10r.

This is a loose translation of *Theorema prima* of the Latin text, and it is claimed that it “may be assumed as demonstrated in the Statik Elements”. It is followed by a Problem: “To find the waighte of any proposed quantity or magnitude of water”. Here the “quantity or magnitude” is understood to be the *volume* of the water. The method proposed is to construct a standard object, for example a solid cube with sides of 3 inches, and weigh it in air and in water. The difference between these numbers is then the weight of 27 cubic inches of water. A rather unrealistic example is given, in which the respective weights of the cube are 16 ounces and 4 ounces. so that 27 cubic inches of water weighs 12 ounces. From this result the weight of any given volume of water can be calculated by proportion, for example a cubic foot (1728 cubic inches) weighs 768 ounces. Apart from the obvious practical questions, the result is unconvincing, because 27 cubic inches of water weighs significantly more than 12 ounces (see Section 4).

The Problem is followed by “Definition 2”, which describes explicitly the construction and use of the device by means of which the term *weight in water* is defined. It is a modification of the equal arm balance, adapted so that the object can be balanced while submerged in water, as well as in the usual way. This is what we now call a hydrostatic balance. Finally there is a “Definition 1^{r}“, presumably intended to be a clarification of the terminology, although Definition 1 itself is lacking in the English text. It points out that the cube used in the preceding Problem must be made of a substance that sinks in water, and so the term “solid body” must be understood to refer only to objects having this property. The discussion appears to end at this point, but there may have been a continuation, now missing.

The second of the English folios (f.11r) is shown in Figure 2. It is concerned with the question addressed in *Theorema secunda* of the *Praxis*, but now hydrostatic weighing is used. The algebraic notation is closer to Harriot’s, in that the symbols are lower-case letters, so it is tempting to suggest that this version is based on a Harriot manuscript, but if so the original has not been found. The heading reads

A problem for exhibiting the seuerall waytes of two mettalls contayned in any commixture by waying the same in ayre and water.

Specifically, the problem is to find the amounts of silver and copper in an alloy, a sample of which is given. The author was probably thinking of the sample as a cube with sides of known length, as in the previous folio. The volume of the cube is referred to as the "quantity" of the sample, and its weight in air is denoted by q. Cubes of the same volume, made of pure silver and pure copper, are assumed to have been weighed in air, and their weights are denoted by b and c. If the weight in air of an identical cube of water is denoted by r, the weights of the three metallic cubes in water are therefore $q-r$, $b-r$, and $c-r$. The problem is to find the weight (in air) of the silver in the sample. This is denoted by a, so that the weight of the copper is $q-a$.

The first two lines of the argument are statements about proportion. They are expressed in the form

$$\overset{,}{u} \;.\; \overset{,,}{v} \;.\; \overset{,}{y} \;.\; \overset{,,}{z}$$

meaning (in modern terms) that the ratio of u to v is the same as the ratio of y to z, which implies that $uz = vy$. In the first line the statement is about the weight of silver in air and in water:

$$\overset{,}{b} \;.\; \overset{,,}{b-r} \;.\; \overset{,}{a} \;.\; \overset{,,}{(ba-ra)/b} \;.$$

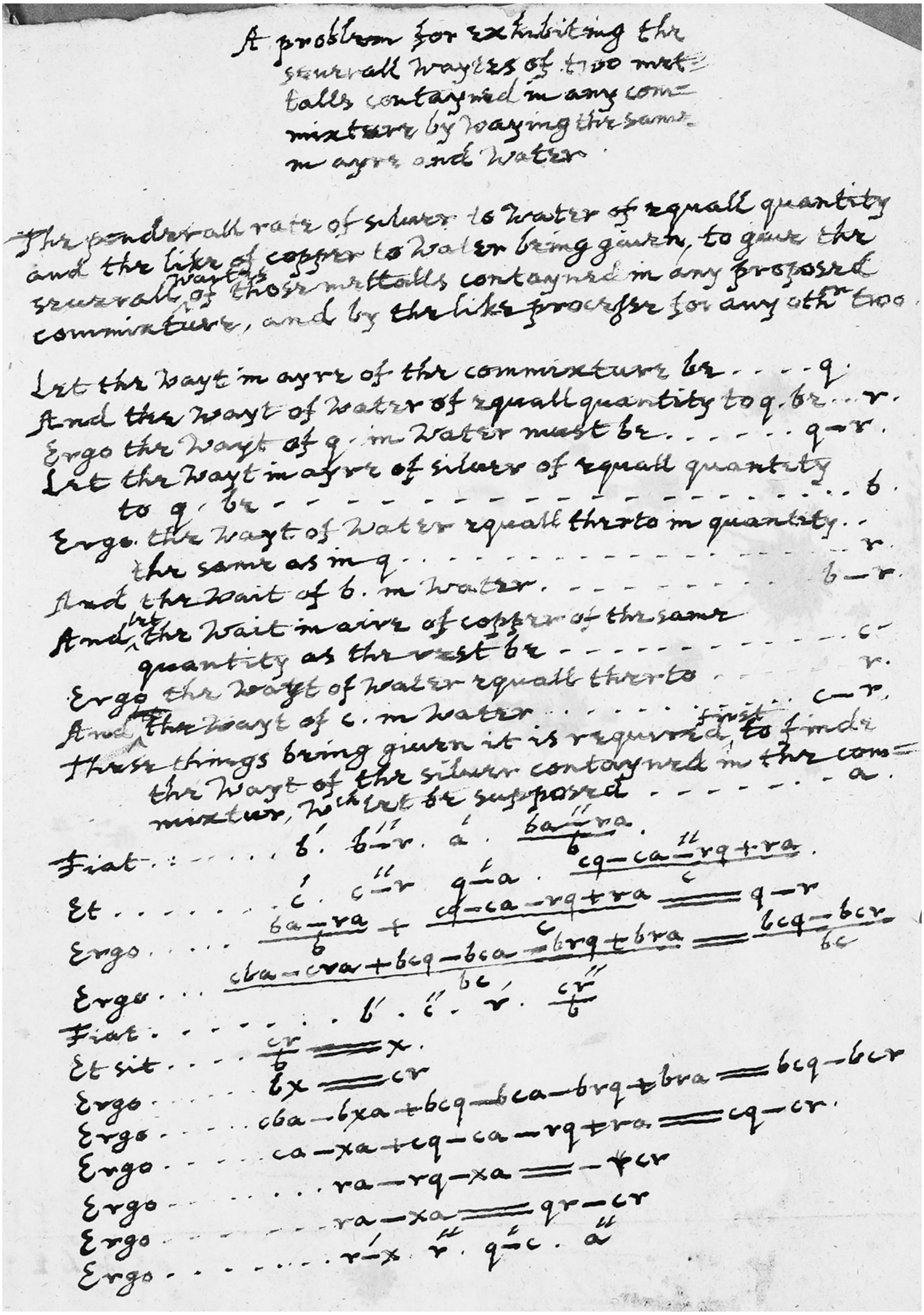
In other words, given that the ratio of the weights of silver in air and in water is as b to $b-r$, the ratio of the weight a of a given amount in air to the weight of that amount in water is as a to $(ba-ra)/b$. Similarly the weight of the copper in water is $(cq-ca-rq+ra)/c$. Since the total weight of the alloy in water is $q-r$, the basic equation follows:

$$(ba-ra)/b + (cq-ca-rq+ra)/c = q-r.$$

This equation appears in the third line of the algebra. The explicit solution

$$a = b(q-c)/(b-c),$$

could now be found by what we call Elementary Algebra, but that is not the path taken here. Instead there is a lengthy diversion, involving more statements of proportion and the introduction of a new symbol $x = cr/b$. The final formula is not stated explicitly; it must be obtained from yet another proportion statement. This method obscures the fact that the result does not depend on r, as is clear from the explicit formula.



A problem for exhibiting the severall wayghts of two mettalls contayned in any commixture by waying the same in ayre and water

The ponderall rate of silver to water of equall quantity and the like of copper to water being given, to give the severall wayghts of those mettalls contayned in any proposed commixture, and by the like processe for any oth^r two.

Let the wayt in ayre of the commixture be . . . . q.
And the wayt of water of equall quantity to q. be . . r.
Ergo the wayt of q. in water must be . . . . . . q−r.
Let the wayt in ayre of silver of equall quantity to q. be . . . . b.
Ergo the wayt of water equall therto in quantity the same as in q . . . . . . r.
And the wait of b. in water . . . . b−r.
And let the wait in aire of copper of the same quantity as the rest be . . . . . . c.
Ergo the wayt of water equall therto . . . . r.
And the wayt of c. in water . . . . . c−r.
These things being given it is required, first to finde the wayt of the silver contayned in the commixture, w^ch let be supposed . . . . . a.

Fiat . . . . . $b.\ b-r.\ a.\ \frac{ba-ra}{b}.$
Et . . . . . $c.\ c-r.\ q-a.\ \frac{cq-ca-rq+ra}{c}$
Ergo . . . . $\frac{ba-ra}{b} + \frac{cq-ca-rq+ra}{c} = q-r$
Ergo . . . . $\frac{cba-cra+bcq-bca-brq+bra}{bc} = \frac{bcq-bcr}{bc}$
Fiat . . . . . $b.\ c.\ r.\ \frac{cr}{b}$
Et sit . . . $\frac{cr}{b} = x.$
Ergo . . . . $bx = cr$
Ergo . . . . $cba-bxa+bcq-bca-brq+bra = bcq-bcr$
Ergo . . . . $ca-xa+cq-ca-rq+ra = cq-cr.$
Ergo . . . . $ra-rq-xa = -cr$
Ergo . . . . $ra-xa = qr-cr$
Ergo . . . . $r-x.\ r.\ q-c.\ a.$
Ergo . . . . . .

Figure 2. Warner’s solution of the alloy problem. NRO IL 3422, Notebook 13, f.11r.

4 The seventeenth-century context

Warner's work on the problem of mixtures was clearly derived from Harriot, but his exposition was clumsy, to say the least. Harriot's own notes rarely offered any explanation of his methods, and, although Warner was trying to provide logical explanations, his efforts were unsatisfactory. The exposition in the *Praxis Statica* was a flawed attempt to mimic the style of Euclid, and his English version failed to exploit the simplicity of Harriot's algebraic symbolism. Furthermore, both attempts relied on unrealistic assumptions about experimental methods. In practice, making an exact cube of any material is not easy, and making a cube of water is extremely difficult.

By the beginning of the seventeenth century an efficient method of hydrostatic weighing was being developed, based on what we now call *specific gravity*, the ratio of the weight of any given volume of a substance to the weight of the same volume of water. For example, in the algebraic notation of Section 3, the specific gravity of silver is b/r. The fact that this is a dimensionless number avoids the complications created by the fact that experimenters in different places use different units of weight and volume. For the alloy problem, the method requires only the experimental determination of the weight of the alloy in air (W_1) and in water (W_2). The specific gravities of silver (B) and copper (C) can be read from a table, and the result is then obtained from the formula, since $q = W_1$, $r = W_1 - W_2$, $b = (W_1 - W_2)B$, and $c = (W_1 – W_3)C$. Indeed, in the course of his experiments on density Harriot had compiled lists of specific gravities, culled from the publications of contemporary scientists, but Warner appears to have overlooked this material.

The scientific work on metals was driven by the interest in the coinage of gold and silver, which represented the economic wealth of the nation. Harriot's experiments on the coinage were carried out in 1604 and 1605, when king James was unifying the coins of his kingdoms. Precise analysis of the composition of coins was essential, and Harriot's experiments revealed several anomalies: for example, a 'gold' coin was likely to contain not only silver (as was commonly assumed), but also a significant amount of copper.[13]

Because there was an international trade in gold and silver, large profits could be made by working the exchanges, and this led to numerous controversies about financial and economic matters throughout the early Stuart period.[14] Francis Bacon (1561-1626) made some measurements of specific gravity by the hydrostatic method, but his results were later dismissed as "very imperfect".[15] A more extensive programme of experiments was carried out in the 1640s by a group of mathematical practitioners. The group included John Reynolds of the Mint, who was regarded as a leading authority on weights and measures at that time.[16] The results of the experiments were published by John Wybard, who wrote that Reynolds had shown him "a cube-inch of fine iron" which (he claimed) had been kept in the Tower since the time of Henry VII in a velvet case: its weight was 4.169 ounces troy.[17] After Reynolds' death in 1665 many of his results on specific gravity were communicated to the Royal Society by Francis Smethwick, and some were eventually published in 1748.[18]

These matters were very much alive in the Interregnum, when the Council of State was greatly concerned about the state of the English coinage. On 31 December 1649 the Council considered evidence from John Reynolds on the "fluctuation of the coin", and shortly afterwards Reynolds sent them a paper by Henry Robinson.[19] In 1651 tables of the weight and value of foreign coins were supplied by the officers of the Mint, led by Reynolds.[20] Many petitions about the management of the coinage were received, foremost among them being those from Thomas Violet, a most unreliable source.[21] Another petitioner was David Ramsay, who was to play a part in the NRO story.[22]

5 Charles Thynne and the Isham connection

Charles Thynne was the third son of Sir John Thynne, the builder of Longleat in Wiltshire. Charles was elected MP for Lymington in 1614 and was engaged in various commercial enterprises, such as the production of salt for the Navy. No official record of his interest in monetary matters has been found, but the NRO collection of Walter Warner's papers is interspersed with those of Thynne. Most notably there is a long petition to the king, which contains suggestions for improving the state of the nation by legislation about the coinage and the exchanges.[23]

There is also a collection of Warner Papers in the National Archives (TNA). They are concerned with the family and business affairs of George Warner, a nephew of Walter Warner, and a merchant based in London. His father (Sylvester) and another uncle (also George) owned estates in Warwickshire, where George senior was High Sherriff in 1641-42. He seems to have been the dominant figure in the family. Several items listed in the TNA catalogue mention Walter Warner and his mathematical works, and these will be explored elsewhere. But one document is particularly relevant to our present topic.[24] It is catalogued as a letter from Charles "Thyme" to George Warner junior, but on examination it is clear that the letter was from Charles Thynne, and that it was actually a complaint about the conduct of George junior, addressed to George senior. It was written shortly after Walter's death in March 1643. No details of the funeral are mentioned in the letter, but its contents are consistent with John Pell's critical account of the event.[25]

Thynne's letter reveals many details of his long friendship with Warner, and the circumstances by which the papers by Thynne and Warner came to be part of the Isham Collection at the NRO. Thynne complains to George Warner senior that George junior has not given him certain books and papers, contrary to Walter's wishes, and asks George senior to instruct his nephew to give them up. He justifies his claim to be the rightful owner at great length, and reveals several hitherto unknown facts about Walter's life and death. The letter begins with an account of Walter's final days, when Thynne visited him regularly and urged him to make a will, apparently without success. Thynne says that they had been close friends for many years, and he lists some of the common interests that they discussed. These included Alchemy and "Monies, Mynt and Exchanges". He says that his own interest in the coinage stemmed from the fact that the king had promised that he would be a Mint Master. He claims that Walter wished him to have all the books and papers relevant to their common interests; however, the "Calculated Tables" should go to the Warner family, who were going to publish them. This is presumably a reference to Warner's tables of antilogarithms, which had been arranged and extended by John Pell in the years 1640-42, but were never published.[26] Contrary to Walter's wishes, his servant had given Thynne only a few papers, saying that the "cheef writings" were locked away. After some negotiations, the disputed material had been put into the hands of a Captain Nelson. The letter ends at this point, and the outcome of the dispute is not recorded.

The existence of the Thynne/Warner archive at the NRO confirms that Thynne did possess a large number of Walter's papers, since his signature is inscribed on many of them, including the Notebooks. It is likely that some of these documents were already in his possession when Walter died, and he may well have acquired more as a result of his letter of complaint,

How did these documents find their way to the NRO? The key figure is Sir Justinian Isham of Lamport Hall in Northamptonshire. He was a well-educated man, and a keen collector of manuscripts. The Isham family collection of over ten thousand items dating back to the 16th century was acquired by the NRO in the years 1930-1974, and the existence within it of Walter Warner's papers was first observed by Timothy Raylor in 1997.[27] The earliest known evidence of a link between Thynne and Sir Justinian Isham is a letter (IC 305) dated 28 November 1651. Thynne reminds Isham of an earlier meeting when they had discussed a project, and he raises the question of 'when we should set up our stills'. The nature of the project is not specified, but there is a reference to "a gentleman whom I conceive to be very skilfull in such things". Thynne says he will ask this gentleman to send Isham some evidence of his expertise.

A letter (IC 306) from the man himself reveals his identity: he was David Ramsay, a fascinating character with wide-ranging interests in science and technology.[28] Ramsay is mainly remembered as the clockmaker to King James and King Charles, but among his many other activities he obtained several patents for inventions.[29] In 1630 he was granted one for "Separating gold and silver from tin, lead, and copper, and for that purpose to set up engines and instruments." In 1638, jointly with Dud Dudley, he obtained a patent which gave him the right to mine and refine metals of all kinds. Ramsay's nephew, John Dury, belonged to the group known as the Hartlib Circle, and Hartlib's *Ephemerides* contains a note of the joint patent, as well as several reports from Dury of his uncle's subsequent attempts to exploit his rights.[30] These efforts appear to have been unsuccessful, probably due to the unsettled conditions of the Civil War, but Ramsay persisted. On 11 February 1651 he submitted a petition to the Council, which was referred to the Mint Committee.[31] Sadly the records of this body are lost.[32] Ramsay's letter to Isham (IC 306) was accompanied by some documents, but these too are now missing. Nevertheless it is reasonable to assume that the documents were concerned with the processing of precious metals. A later letter (IC 293) from Thynne urged Sir Justinian to invest in the project, saying that each 100 pounds invested would surely yield 500 pounds.

It seems that Isham was not persuaded, and nothing came of the project. In fact Thynne died in October 1652, leaving a short will in which there is no mention of his papers.[33] His executors were Jane Dove, a spinster, and Sir James Thynne, his nephew and master of Longleat. If they had to dispose of Thynne's papers they would have been aware of his link with Isham and regarded him as a suitable recipient. It is also possible that Thynne himself gave the papers to Isham before he died. In any case we know that Isham had them in 1653, because Samuel Hartlib recorded that "Sir Justinian Isham hath gotten all the MS mathematical of Warner and Tin as himself showed them to Mr Pell".[34]

6 Conclusion

The documents discussed here constitute a very small fraction of Warner's unpublished papers. A recent trawl of the material at the NRO has revealed that the papers are more extensive than was previously thought, and a full assessment of them will be a major project. At first sight, the documents confirm the view that Warner was a faithful follower of Harriot, although his attempts at exposition were flawed. He might have made an important contribution to mathematics with his tables of antilogarithms, but these were never published.

In a wider context, Warner was influential because his contemporaries regarded him as the means by which Harriot's discoveries might be published and used for practical purposes. For example, Warner's notes on hydrostatics show that scientific research in the first half of the 17th century could be motivated by problems in the field of money and finance. Indeed, it is likely that much information about issues of national significance remains hidden in the archival undergrowth.

Notes

1. For general surveys of the life and work of Harriot see the biographies by Shirley (1983) and Arianrhod (2019).
2. Knorr, *Ancient Sources*; Moody and Clagett, *Medieval Science.*
3. For Harriot's sources, see Biggs, "Harriot on the Coinage" and Clucas, " Curious Ways".
4. Shirley, *Harriot*, 400. The reference is to Ghetaldi, *Promotus Archimedis*, 1603.
5. Stedall, *Discourse,* 100. Stedall also cites evidence that Warner was responsible for preparing the *Artis analyticae praxis* for publication, although his name does not appear in the book.
6. The NRO items mentioned in this article are as follows. IL 3422 is a set of Warner's Notebooks, one of which (Number13) is discussed in Sections 2 and 3. Charles Thynne's writings on coinage and exchange (Section 5) can be found in IL 3344, IL 3393, IL 3421, and IL 3442. The letters mentioned in Section 5 are IC 293, IC 305, and IC 306.
7. Moody and Clagett, *The Medieval Science of Weights*, 119-227
8. Clucas, "Curious Ways", 326. The scribal copy is part of BL Harleian MS 6754.
9. Malcolm and Stedall, *John Pell*, 358. Pell is critical of a kinsman of Walter, presumably George Warner junior, and claims that "he sent his partner to bury the old man".
10. Halliwell, *Letters,* 65-70.
11. Miller, *That Noble Cabinet*, 46. The transfer was part of the Act by which the British Museum was founded.
12. For the numismatic context, see Biggs, "Without Grains".
13. Biggs, "Harriot on the Coinage", 372.
14. Supple, *Commercial Crisis*, Chapter 8.

15. Davies, “Tables”, 422.
16. Biggs, “John Reynolds”, 24
17 Wybard, *Tactometria*, 226.
18. Davies, “Tables”, 424-5, 483.
19. *CSPD 1649-50*, 462, 475. Robinson’s interest in the exchanges appears in his 1641 tract *England’s Safety*, printed in Shaw, *Select Tracts*, 46-60 . In the early 1650s he held important positions in the government.
20. The Tables were published by Shaw, *Select Tracts*, 85-100. Reynolds’ results were retained at the Mint, and they were cited by Isaac Newton in his notes on foreign coins: see MINT 00216 and MINT 00243 in the online Newton Project (https://www.newtonproject.ox.ac.uk).
21. *CSPD 1651-2,* 24-25.
22. *CSPD 1651-2,* 140.
23, As note 6. IL3344 contains a reference to King James, which suggests that Thynne’s interest in coinage and exchanges began in that reign.
24. TNA SP 46/189/109.
25. Malcolm and Stedall, *John Pell*, 358. Pell is critical of a “kinsman” of Walter, presumably George Warner junior, and claims that he “sent his partner to bury the old man.”
26. Malcolm and Stedall, *John Pell*, 280-286.
27. Stedall, *Discourse*, 241.
28. Finch et al., “David Ramsay”. This recent biography corrects several errors in previous accounts of Ramsay’s life and work.
29. Woodcroft, *Alphabetical List*, 466.
30. Hartlib, *Ephemerides*, 31/22/40B. See also 1/2/1A for a report from John Dury.
31. As note 22.
32. Challis, *Mint*, 324-325..
33. TNA PROB 11/223/405 and PROB 11/241/49. There are two items because the two executors received separate grants of probate.
34. Hartlib, *Ephemerides*, 28/2/40A.

Acknowledgements

The author thanks the staff of the Northamptonshire Record Office for permission to publish the documents shown in Figures 1 and 2. Andrew North’s work on locating and reviewing the NRO documents has resulted in many new discoveries. The author is also grateful to Stephen Clucas for sharing his notes on some of the NRO material, and to an anonymous referee for suggestions about improvements to this article.

Disclosure Statement

No potential conflict of interest was reported by the author.

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