STRATEGIC TRADE AND INDUSTRIAL POLICY

TOWARDS DYNAMIC OLIGOPOLIES*

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Abstract

In this paper characterise optimal trade and industrial policy in dynamic oligopolistic markets. If governments can commit to future policies, optimal first-period intervention should diverge from the profit-shifting benchmark to an extent which exactly offsets the strategic behaviour implied by Fudenberg and Tirole's "fat cats and top dogs" taxonomy of business strategies. Without government commitment, there is an additional basis for intervention, whose sign depends on the strategic substitutability between future policy and current actions. We consider a variety of applications (to R&D spillovers, consumer switching costs, etc.) and extensions to second-best, revenue-constrained and entry-promotion policies.

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1. Introduction

The theory of strategic trade policy has grown from precocious urchin to mature teenager and is now part of the central canon of international trade theory. Much of its initial appeal came from the fact that it appeared to provide a stronger justification for interventionist measures such as tariffs and export subsidies than does traditional competitive theory. However, as is now well known, the specific policy recommendations of the theory are highly sensitive to changes in assumptions about firm behaviour, entry, and so on. For this reason, possibly a more important contribution of the theory is that it highlights a key aspect of public policy in oligopolistic markets: that governments and firms are likely to differ in their ability to commit to future actions. Thus the desirability of intervention, whether an export subsidy as in Brander and Spencer (1985) or an export tax as in Eaton and Grossman (1986), derives from the government's assumed ability to commit to policies which will remain in force while firms take their decisions on outputs or prices. The optimal policy moves the home firm to the point which it would attain unaided *if* it had a Stackelberg first-mover advantage.

Yet the superior ability of governments to commit is questionable on two counts. First, firms too may have the ability to commit to variables such as capacity, advertising or R&D, which will affect the strategic environment in which policies are set and outputs and prices are chosen in the future. Second, a government's assessment of its own powers of commitment may not be shared by the private sector. With or without a past history of reneging on its commitments, its policy announcements may lack credibility with the firms whose behaviour it is trying to influence. Of course, it may also be the case that the government does not in fact possess the ability to commit to policies some time in the future. These considerations prompt a search for policies which are time consistent.

The purpose of this paper is to consider the implications for strategic trade and industrial policy of allowing firms to make commitments in advance. We do not rule out government commitment altogether: to do so would be unrealistic and would remove any basis for strategic policy. However, we allow for the possibility that governments may be unable to commit far into the future; and also for the possibility that firms may be able to take some decisions (such as investment, R&D, etc.) which have effects over a longer time horizon than that over which the government can commit.

As we shall see, models of dynamic oligopolies with imperfect government commitment introduce two new motives for government intervention in addition to the standard profit-shifting motive familiar from static models. First, in the manner first studied by Spence (1977) and Dixit (1980), firms have an incentive to engage in behaviour which will improve their strategic position vis-à-vis their rivals in the future. This type of strategic behaviour is socially wasteful (even when the rival firm is foreign) and government policy should be addressed towards offsetting it. Second, if the government cannot commit to future policies, firms have an additional incentive to try and influence their future entitlement to assistance, for example, by over-investing in order to qualify for a higher subsidy. The need to restrain the home firm from this strategic behaviour provides the government with an additional motive for intervention in the earlier period.

Aspects of imperfect commitment and dynamic behaviour in models of strategic policy

have been explored in a number of recent papers. Brander and Spencer (1987) consider a model where an incumbent home firm chooses its level of R&D before the government chooses its output subsidy. Carmichael (1987), followed by Gruenspecht (1988) and Neary (1991), explores the implications of allowing firms to set prices before the government chooses its subsidy. Maskin and Newbery (1990) consider the optimal time-consistent import tariff on a natural resource produced by a competitive industry. Goldberg (1995) and Maggi (1996) consider optimal policy in models where firms choose their capacities before the output or price stage. Finally, we have explored some of these issues in models with learning by doing (Leahy and Neary 1994, 1998) and R&D (Leahy and Neary 1996, 1997). However, this literature is fragmented and the general lessons are often obscured by the particularities of individual models. In this paper we present a general framework which nests most of these models and relates them to the taxonomy of business strategies developed by Fudenberg and Tirole (1984).[1]

In the remainder of the paper we address the questions raised here by first considering a general dynamic two-firm multi-period model in Section 1. The model adopts the standard "third-market" framework of Brander and Spencer (1985), abstracting from domestic consumption in order to focus on the strategic interaction between firms and government.[2] Rules for optimal intervention are derived under different assumptions about the ability of firms and government to commit to future actions. These rules are then applied to a series of special models in Section 2. This allows us to consider the implications for policy of alternative assumptions about the dynamic links between markets. Section 3 returns to the general two-period model and shows how it may be extended to deal with second-best problems (where the government can intervene in only one period); with divergences between the marginal social valuation of corporate profits and subsidy revenue; and with cases where the home firm makes insufficient profits at the first-best optimum, so policies must be supplemented by entry-promotion considerations. The Appendices prove some key results and give details of the solution in an important special case.

2. Optimal Intervention in a General Model of Dynamic Oligopoly

2.1 The Firms' Decision Problem

We consider a market with two firms, denoted "home" and "foreign", which compete over an indefinite (though finite) number T of time periods. In each period t, each firm takes an action, choosing the value of some decision variable, a_t for the home firm and b_t for the foreign firm. This specification allows for many alternative types of oligopolistic interaction: in each period the decision variables might be output, price, R&D, etc. Each firm seeks to maximise the present value of its profits, so the decision problem of the home firm can be written as:

$$\operatorname*{Max}_{a} \boldsymbol{p}\,(a,b,s) \;=\; R(a,b) + S(a,b,s)\;,$$

<div style="text-align:right">1</div>

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[i] Laussel and Montet (1994) use the Fudenberg and Tirole taxonomy in a different manner to interpret the results of strategic trade policy in one-period models.

[2] One rationale for this is that all output is exported. Alternatively, we can assume that some output is domestically consumed, that home and foreign markets are integrated and that domestic consumer surplus does not enter the welfare function.

where *a* and *b* are the vectors of the home and foreign firm's actions respectively. Here **p** denotes the present value of the firm's profits, equal to the sum of the present values of its gross profits (i.e., sales revenue less total costs) *R* and its subsidy income *S*. Each of these components is a twice differentiable function of its own and its rival's actions; in addition, *S* depends on a vector of subsidy rates chosen by the home government, *s*. In most cases, the subsidy is directly related to the firm's decision variable, so $S^t = s_t a_t$ and $S_t/MS^t/Ma_t = s_t$, where S^t is subsidy income in period *t*. An exception is the case of an output subsidy in Bertrand competition: the firm's decision variable a_t is now its price, but its subsidy income S^t equals $s_t q_t$, where q_t is the period-*t* demand function which in general depends on all current and past actions (prices). In this case $S_t = s_t M q_t/Ma_t$, so the derivative of S^t with respect to a_t has the opposite sign to s_t.[3] As for the foreign firm, its decision problem is similar to 1 but with no subsidy terms; we assume that the foreign firm is not assisted by its government:

$$\text{Max}_b \ R^*(b,a). \qquad\qquad \textbf{2}$$

The basic results of the paper are most easily derived using the multi-period specification in 1. However, in order to interpret them and to apply them to many examples it is desirable to focus on a two-period special case. Assuming that firms compete over only two periods, labelled "1" and "2", allows us to write the home firm's gross profits and subsidy income functions as follows (where **r** denotes the discount factor):

$$R(a,b) \ = \ R^1(\ a_1,b_1\ ) + \textbf{r}\ R^2(\ a_1,b_1,a_2,b_2\ ) \ , \qquad \textbf{3}$$

$$S(a,b,s) \ = \ S^1(\ a_1,b_1,s_1\ ) + \textbf{r}\ S^2(\ a_1,b_1,a_2,b_2,s_2\ ) \ . \qquad \textbf{4}$$

This specification draws attention to the dynamics of the model. While first-period profits depend only on variables in that period, second-period profits are directly affected by the actions taken in the previous period. The influence of the firm's own action a_1 arises from dynamic linkages such as investment in capacity or learning by doing; the influence of the foreign firm's action b_1 reflects an externality such as R&D spillovers.

2.2 Firm Behaviour under Alternative Assumptions about Commitment

The second respect in which the model is dynamic is in its assumptions about commitment. As in our earlier work (Leahy and Neary, 1997, 1998), this has a number of aspects. We assume throughout that the government is able to commit *intra*-temporally in all periods, setting each subsidy s_t before the firms choose their actions in the same period a_t and b_t. However, it may or may not be able to commit *inter*-temporally, setting the subsidy s_t before the firms choose their actions in earlier periods $a_{t!i}$ and $b_{t!i}$. This leaves a number of alternative assumptions about move order and we concentrate on three central cases (which are the only ones that can arise in the two-period model). Consider first cases where the government can commit in

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[3] A simpler approach, following Brander (1995), is to assume that the subsidy is always directly related to the firm's decision variable, so when this variable is its price, a subsidy to it is equivalent to an output *tax*. However, this equivalence between price and output subsidies does not extend to the case of Sequence Equilibrium discussed below.

period 1 to all future subsidies. Subject to this, we refer to the case where firms can commit in period 1 to all their future actions as "Full Commitment Equilibrium" or "FCE"; and to the case where firms cannot commit to any future actions as "Government-Only Commitment Equilibrium" or "GCE". Finally, we refer to the case where no agents can commit in advance, and in particular where the government cannot commit in advance to any future subsidies, as "Sequence Equilibrium" or "SE". For each of our three sets of assumptions about commitment the solution concept is that of Nash subgame perfect equilibrium. The values of all variables will differ in general between the three equilibria, although it is convenient to derive the first-order conditions in a general form which applies in all three cases. Finally, we assume until Section 3.3 that a unique interior equilibrium exists in each case, so issues of exit and entry do not arise.

The implications of the different equilibrium concepts can be seen more concretely by considering the behaviour of the home firm, which chooses the value of a to set its marginal profitability to zero. In general the implies the following:

$$\left(\frac{d\boldsymbol{p}}{da} \right) = \boldsymbol{p}_{a'} + \boldsymbol{p}_{b'} \frac{db}{da} + S_{s'} \frac{ds}{da} = 0 \ .$$

5

Here a prime denotes the transpose of a vector, so 5 represents T first-order conditions, one for each period; and subscripts to functions denote partial derivatives. (We are able to replace $\boldsymbol{p}_s$ by S_s in the final term, because R is independent of s from 1.) Each of the three terms in 5 arises in SE, but only the first two arise in GCE and only the first in FCE. The precise form which these terms take is the focus of the remainder of Section 1.

2.3 Welfare and the Optimal Subsidies

This completes our discussion of the home firm. Turning to the government we assume that all output is exported and (until Section 3.2) that the marginal social cost of funds is unity. Hence welfare equals the firm's profits net of subsidy payments:

$$W\,(a,b) \ = \ \boldsymbol{p}(a,b,s) - S(a,b,s) \ = \ R(a,b) \ .$$

6

This welfare function depends on both home and foreign decision variables (though not directly on the subsidies) and we wish to find the values of the subsidies which will maximise it. To solve this problem with two targets (a and b) and one instrument (s), it is useful to think of the government's choice of subsidies as giving it direct control over the home firm's actions a, while it controls the foreign firm's actions indirectly. Of course, the ways in which this control is exercised, and so the details of how equation 6 is maximised, depend on the government's powers of commitment. Nevertheless, it is insightful to begin with a general formulation which covers all cases.

Totally differentiating 6 gives a necessary condition for welfare maximisation:

$$dW \ = \ R_{a'}\,da + R_{b'}\,db = 0 \ .$$

7

To solve for the optimal subsidies we first replace the coefficient of da using the home firm's

first-order condition 5 (exploiting the fact that $p_a = R_a + S_a$). As for the second term, it can be eliminated by using the solution to the foreign firm's first-order conditions. These take the following form:[4]

$$\left(\frac{dR^*}{db} \right) = (R_b^*)' + (R_a^*)' \frac{da}{db} = 0 .$$

8

We can think of the government as solving these equations for *generalised* reaction functions, which express the foreign firm's actions as functions of *all* of the home firm's: $b = B(a)$. In differential form, this implies:

$$db = B_a\, da .$$

9

We can now use these differentiated reaction functions to eliminate db from 7, and solve for the

$$S_{a'} = R_{b'} B_a - p_{b'} \frac{db}{da} - p_{s'} \frac{ds}{da} .$$

11

$$S_{a_1}^1 = -r\, S_{a_1}^2 + \{ R_{b_1}^1 + r\, R_{b_1}^2 \} B_{11} + r\, R_{b_2}^2 B_{21} - p_{b_2} \frac{db_2}{da_1} - p_{s_2} \frac{ds_2}{da_1} ,$$

10

$$S_{a_2}^2 = r^{-1} \{ R_{b_1}^1 + r\, R_{b_1}^2 \} B_{12} + R_{b_2}^2 B_{22} .$$

12

optimal subsidies:

This equation is the central result of the paper. For some purposes it is helpful to write it in full for the two-period case. This gives:

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[4] In SE the foreign firm also anticipates the effects of its actions on the home government's future subsidy. However, this only affects its profits through the home firm's future actions, so there is no *ds/db* term in 8.

Recall that S_t is the partial derivative of the subsidy revenue function S^t with respect to a_t; in most cases it is simply s_t, except in the case of an output subsidy in Bertrand competition when it equals $s_t Mq_t/Ma_t$ and so is *inversely* related to the subsidy. As for the cross-term S_1 in 11, it represents the direct or non-strategic effect of a change in a_1 on future subsidy revenue.[5] In most cases this term is zero; an exception is the case of an output subsidy in Bertrand competition when it equals $s_2 Mq_2/Ma_1$ and so reflects intertemporal effects such as switching costs in demand, discussed in Section 2.5 below. When the term is non-zero the two equations have to be solved recursively, with the optimal value for s_2 first calculated from 11 and then substituted into 11 to calculate the optimal value of s_1.

2.4 Optimal Policy with Full Commitment

Consider first the FCE case where both government and firms (in that order) commit in period 1 to their actions in all periods. As already noted, the final two terms in 11 (and hence in 11) do not arise in FCE. Hence, irrespective of the number of periods, the expression for the optimal subsidies takes a particularly simple form:

$$S_{a'} = R_{b'} B_a .$$
13

Both the form of this equation and the rationale for intervention which it provides are identical to those in one-period models. Algebraically, 13 states that the optimal subsidy (or, in Bertrand competition, the optimal subsidy times the slope of the demand function) should equal the cross-effect of foreign actions on home revenue (R_b) times the slope of the foreign reaction function (B_a). (With many periods all of these terms should be interpreted as vectors or matrices where appropriate.) In terms of economic intuition, the justification for intervention is that, because of its superior commitment power, the government is able to move the foreign firm along its reaction function to the Stackelberg equilibrium, something which the home firm itself cannot credibly do. We will follow standard practice and refer to this as the "profit-shifting" motive for intervention.

Of course, the simplicity of 13 arises from the compact matrix notation. When we consider the elements in this expression period by period, matters appear more complex, as equations 11 and 11 show. (We consider here only the pure profit-shifting effects, represented by terms involving the B_{ij} derivatives. All other terms are discussed elsewhere.) To interpret these terms, we introduce the distinction between *inter-* and *intra-temporal* profit shifting. In each equation there is one term, $R_t B_{tt}$ *(t=1,2)*, all of whose components relate exclusively to the same period. These terms represent the standard *intratemporal* profit-shifting motive: if there are no links between periods all other terms vanish (since $R_j=B_{ij}=0$ for $i...j$) and both equations collapse to $S_t=R_t B_{tt}$ *(t=1,2)*. This yields the familiar results of static strategic trade policy.[6] In addition, because of the dynamic structure of the model, there is an *intertemporal*

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[5] Following Fudenberg and Tirole (1984), we call the first term in 5 the "direct" or "non-strategic" effect of the firm's actions. Of course, the firm is always a Nash player, so strategic considerations arise even for this term; however, the label "non-strategic" is convenient and hopefully unambiguous.

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[6] When competition is Cournot: S_t equals s_t; R_t is negative provided goods are Antonelli substitutes in demand; and B_{tt} is negative provided outputs are strategic substitutes. Hence an export subsidy is optimal, as in Brander

profit-shifting motive in both periods, represented by cross-terms such as R_1 and B_{12} in 11 and 11. Calculating all the components of S_a in any particular application is likely to be difficult. However, the difficulty is no more and no less than that needed to sign any matrix comparative statics expression.[7] Summarising the results of this sub-section:

Proposition 1: In FCE, the formula for the optimal subsidies is a matrix generalisation of the static profit-shifting formula, reflecting inter- as well as intra-temporal rent-shifting.

2.5 Optimal Policy with Strategic Competition by Firms

Consider next the GCE case, when the government can still commit in advance to future subsidies but firms cannot commit in advance to future actions and so engage in strategic behaviour against each other. The home firm's strategic behaviour is represented by the second term in its first-order condition 5. As the second term in 11 shows, optimal policy requires that this strategic effect should be exactly offset.

One case where this has dramatic implications is where the home firm and the home government have the same ability to commit (both inter- and intratemporally) and so the same ability to manipulate the foreign firm. In that case, $B_a = db/da$, and equation 11 shows that the optimal subsidy is zero.[8] This confirms from a different perspective the points made in the last sub-section. If the home firm can commit to the same extent as the home government, then it acts as a Stackelberg leader on its own, and the rationale for strategic trade policy disappears. By contrast, we concentrate on the case where the home firm is always a Nash player against the foreign firm, so all elements on and above the principal diagonal of the matrix db/da are zero; and where the home government always has superior commitment power (at least intratemporally), so some or all of the corresponding elements in the matrix B_a are non-zero.

To obtain concrete results we must now specialise to the two-period case, where the key term is that involving db_2/da_1 in 11. This term has been extensively studied, most comprehensively by Fudenberg and Tirole (1984) and Tirole (1988, Chapter 8), and we follow their taxonomy here.[9] The expression db_2/da_1 in this term is the foreign firm's period-2 response to a change in a_1, as anticipated by the home firm. It arises from the solution of the two second-period first-order conditions. That for the foreign firm implies a period-2 reaction

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and Spencer (1985). By contrast, when competition is Bertrand: S_t equals s_t q_t / a_t; R_t is positive provided goods are Hicksian substitutes in demand; and B_{tt} is positive provided prices are strategic complements. Hence an export tax is optimal, as in Eaton and Grossman (1986).

[7] To see this, we calculate B_a explicitly, using the first-order conditions for the foreign firm (i.e., equation 8 specialised to the FCE case): $R(b,a)=0$. Totally differentiating and substituting in 13 gives: $S_a = R_b (R_b)^{-1} R_a$. (Compare, for example, the single-period Cournot result given by Brander (1995), equation (3.17).) The only general restriction on this formula is that R_b must be negative definite from the foreign firm's second-order condition. However, other restrictions may be imposed under appropriate additional assumptions. For example, if in every period the foreign firm's action is a strategic substitute for *every* action of the home firm, then all the elements of R_a are positive.

[8] To be precise, when $B_a = db/da$, the expression for the optimal subsidy reduces to $S_a = S_b B_a$. This implies zero optimal subsidies in all cases.

[9] Since there are two incumbent firms, the Fudenberg-Tirole taxonomy we follow is that for entry accommodation rather than entry deterrence. We consider entry-promotion policies in Section 3.3.

function, $b_2 = \mathbf{y}(a_1, b_1, a_2)$. Differentiating this, the full response of b_2 to a_1 may be decomposed as follows:

$$\frac{db_2}{da_1} = \mathbf{y}_{a_1} + \mathbf{y}_{a_2} \frac{da_2}{da_1} \, ,$$

14

where the derivative da_2/da_1 arises from the joint solution of *both* firms' period-2 first-order conditions. The partial derivative $\mathbf{y}_{a1}$ in 14 reflects an intertemporal externality between the two firms.   Otherwise the right-hand side term is as in Fudenberg and Tirole and its contribution to profits may be broken into two components. First is the slope of the foreign firm's reaction function in the second period, $\mathbf{y}_{a2}$: this is negative when second-period actions are strategic substitutes, and positive otherwise. Second is whether or not "investment", in the sense of an increase in a_1, makes the home firm "tough", in the sense of reducing the foreign firm's period-2 profits.[10] The resulting taxonomy, which is illustrated in Table 1, is now well-known, although we shall see in the next section that its detailed implications for policy are not immediately obvious.

Table 1
Taxonomy of Strategic Effects and their Implications for Optimal Policy

		Investment Makes Home Firm:	
		Tough: $R^{*2}_{a_2} \dfrac{da_2}{da_1} < 0$ 1	Soft: $R^{*2}_{a_2} \dfrac{da_2}{da_1} > 0$ 2
Slope of Reaction Functions in Second Period	Strategic Substitutes $\mathbf{y}_{a2} < 0$	"Top Dog": "Restrain" R&D+C: tax a_1 LBD+C: tax a_1	"Lean and Hungry": "Fatten" NR+C: subsidise a_1
	Strategic Complements $\mathbf{y}_{a2} > 0$	"Puppy Dog": "Encourage" R&D+B: subsidise a_1 NR+B: tax q_1	"Fat Cat": "Exercise" LBD+B: subsidise q_1

C:	Cournot (Output) Competition
B:	Bertrand (Price) Competition
R&D:	Research and Development

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[10] Ignoring $\partial R^{*2}/\partial a_1$, the direct effect (if any) of a_1 on R^{*2}, the effect of a_1 on the foreign firm's profits is: $dR^{*2}/da_1 = R\{da_2/da_1\}$. By analogy with Fudenberg and Tirole, we assume (plausibly) that the cross-effects on profits in the second period, $\mathbf{p}_2$ and R, have the same sign. Hence "toughness" is equivalent to a positive value for da_2/da_1 as required.

LBD: Learning by Doing
NR: Natural Resources

For the present, note that the second term in equation 11 for the optimal period-1 subsidy exactly offsets the Fudenberg-Tirole strategic effect in the home firm's first-order condition 5. Heuristically, strategic behaviour by the home firm consumes real resources. Since (until Section 3.2) the marginal social cost of funds is unity, it is more efficient for the government to restrain the home firm from this wasteful strategic behaviour while simultaneously shifting rents optimally just as in FCE. Summarising:

Proposition 2: In GCE, the formula for the optimal period-1 subsidy contains an additional term, which exactly offsets the Fudenberg and Tirole strategic effect.

Naturally, Fudenberg and Tirole's "cats and dogs" taxonomy of business strategies suggests an "animal training" taxonomy of optimal policy responses. We explore this in more detail when we consider applications in the next section.

2.6 Optimal Policy without Government Commitment

Finally, consider the Sequence Equilibrium case, where the home government cannot commit in advance to its future subsidies and so the home firm has an incentive to play strategically against it. This incentive is represented by the new third term in equation 5. The government in turn adjusts its optimal subsidy to offset this term, as the third term in 11 shows. The other terms in equations 11 and 11 also continue to apply in SE, but because of the sequential nature of actions by all agents, the interpretation of the B_{ij} coefficients is rather different.

To see how the subsidies are set in SE, consider the government's problem at the beginning of period 2. With a_1 and b_1 already determined, it faces a standard static problem and the optimal subsidy is given by the static rent-shifting formula:

$$S^2_{a_2} = R^2_{b_2} \mathbf{y}_{a_2} .$$

15

This can be thought of as a special case of 11, with B_{12} set equal to zero and B_{22} equal to $\mathbf{y}_{a_2}$, the slope of the foreign firm's period-2 reaction function. Of course, the individual terms in 15 depend on past decisions and so it can be solved for the period-2 optimal subsidy as a function of the period-1 actions of both firms:

$$s_2 = s_2(a_1, b_1) .$$

16

Both firms take this into account in choosing their first-period actions, which explains the *ds/da* term in 5.[11]

Finally, the government anticipates this behaviour in setting its period-1 subsidy, which

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[11] In SE the foreign firm also anticipates the effects of its period-1 action on the home government's period-2 subsidy. However, this only affects its profits through the home firm's period-2 action, so there is no *ds/db* term in 8.

explains the final term in 11.[12] This term exactly offsets the strategic behaviour by the home firm, in varying a_1 with the intention of manipulating the period-2 subsidy. This term is negative (since p_{s2} is positive) if and only if an increase in the home firm's period-1 action tends to raise the period-2 subsidy; i.e., if and only if s_2 is a *strategic complement* for a_1. Summarising:

Proposition 3: In SE, the formula for the optimal period-1 subsidy contains an additional term, which exactly offsets the strategic behaviour of the home firm vis-à-vis the home government, tending to lower S_1 if and only if the period-2 subsidy is a strategic complement for the home firm's period-1 action.

2.7 Welfare Comparisons between Equilibria

So far, we have been able to isolate the factors which determine the optimal subsidies, and, as the next section will show, they can be signed in many applications. It would also be very desirable to give a welfare ranking of the different equilibria, but this is not possible in general. However, it can be done in one special case, where the foreign firm has no intertemporal links:

Proposition 4: If the foreign firm has no intertemporal links, the real equilibrium (including the level of welfare) is the same in all three cases. Only the subsidies differ.

This proposition is easily proved by substituting for the optimal subsidies 11 into the home firm's first-order conditions 5. This gives:

$$R_a\{a, B(a)\}' + R_b\{a, B(a)\}' B_a(a) = 0 ,$$

<div align="right">17</div>

which is the Stackelberg equilibrium condition in matrix form. A sufficient condition for this to be invariant to changes in the assumptions about move order is that the matrix B_a is diagonal, in other words, that the foreign firm has no intertemporal links.

3. Applications

Equation 11, and its application to the two-period case in equations 11 and 11, is the key result of the paper. In this section we consider its implications in some interesting special cases.[13]

3.1 Investment in Capacity or R&D

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[12] Of course, the rent-shifting coefficients are also very different. For example, B_{21} takes a relatively simple form since it can be calculated directly from the foreign firm's period-2 reaction function: $B_{21}=y_{a1}+y_{b1}\{db_1/da_1\}$. The externality term involving y_{a1} enters positively here and negatively in the expression for db_2/da_1 in 11. However, these are the only terms which cancel in general.

[13] Shapiro (1989, Section 4.4) gives a more comprehensive review of applications of two-period oligopoly games, though he does not discuss policy issues.

The simplest type of intertemporal linkage to which our general approach may be applied is that of investment where all the benefits accrue to the firm which undertakes it. In this model, each firm's first-period decision variable is its level of investment, while its second-period decision variable is either output or price. The home firm's revenue functions simplify to:

$$R^1(a_1) = -C^1(a_1), \qquad R^2(a_1,a_2,b_2) = \hat{R}^2(a_2,b_2) - C^2(a_1,q_2), \quad \mathbf{18}$$

where $R^2(a_2,b_2)$ is sales revenue in period 2. The simplicity of this case arises from the fact that firms do not compete directly in the first period, and so the home firm's revenue functions do not depend directly on the foreign firm's level of investment, b_1. The home firm's period-1 revenue function is simply the negative of its investment cost function $C^1(a_1)$; while in the second period investment lowers production costs $C^2(a_1,q_2)$ at a given level of output q_2 and so raises revenue: $R_1 = !C_1 > 0$.[14] As a result, both R_1 and R_1 are zero, and so the terms in parentheses in equations 11 and 11 are zero. Furthermore, the foreign firm's first-order conditions 8 do not depend directly on the home firm's level of investment in FCE, so the coefficient B_{21} in equation 11 is also zero.[15] The implications for the optimal subsidies are immediate: the second-period subsidy reflects intratemporal profit-shifting solely, while in FCE there is no profit-shifting motive whatsoever in the expression for the first-period subsidy.

So, if firms do not invest strategically (i.e., in FCE), there is no justification for taxing or subsidising investment. And, whether they do or not, the optimal second-period intervention has the same form as in models without investment.

The case where firms behave strategically and second-period competition is Cournot has been considered by Spencer and Brander (1983) and falls into the upper left-hand box of Table 1. Higher investment makes the home firm tough (i.e., it reduces the foreign firm's profits) and outputs are "normally" strategic substitutes. Hence, the home firm has an incentive to over-invest relative to the social optimum and the optimal period-1 policy is an investment *tax*. Extending Fudenberg and Tirole's taxonomy, the government should *restrain* this "top dog". Of course, this need not mean that investment is lower overall, when the full effect of intervention is taken into account. Figure 1 illustrates (under the simplifying assumption that the foreign firm does not invest). Without intervention, strategic behaviour by the home firm leads to inefficient over-investment along OKN rather than the efficient locus OK, which shifts its output reaction function outwards from H to HN. The optimum GCE policy package of an investment tax and an export subsidy restores efficient investment, though (at least with linear demands) investment and output are still higher.

By contrast, if period-2 competition is in terms of prices, the second-period reaction functions are "normally" upward-sloping. Investment still makes the home firm tough but now it has an incentive to *under*-invest, leading to higher prices for both firms. To counteract this

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[14] Recall that q_2 is period-2 output, equal to a_2 in Cournot competition and $q_2(a_2,b_2)$ in Bertrand competition. This specification of costs allows for many interesting special cases, including that of marginal cost independent of output: $C^2(a_1,q_2)=c(a_1)q_2$, $c? <0$; and the "plant design" model of Vives (1989) which allows for a trade-off between fixed and variable costs: $C^2(a_1,q_2) =c(a_1)q_2+a_1q$, $c? <0$.

[15] The same is not true in GCE: since a_2 depends on a_1, so also does da_2/db_1. However, since this effect works only through changes in the curvature of the home firm's period-2 reaction function, it is unlikely to be very important. For example, it vanishes in the case of Cournot competition if marginal costs are independent of output.

behaviour, the optimal policy is an investment *subsidy*: this "puppy dog" should be *encouraged*. In this case, the net effect on investment is ambiguous. Figure 2 illustrates one possible outcome.

The considerations discussed so far apply only to the home firm's strategic behaviour against its foreign rival. In SE it also behaves strategically against the home government, to an extent determined by the sign of ds_2/da_1. Appendix A.3 derives an exact expression for this term and shows that it is presumptively positive when second-period competition is Cournot and negative when it is Bertrand.[16] Hence in Cournot competition the future subsidy is a strategic complement for the home firm's investment. As a result, the home firm has a further incentive to over-invest in order to obtain a higher subsidy, and so the government has a further motive to tax it in period 1 in order to restrain it from this socially wasteful behaviour. A similar chain of reasoning, but with opposite implications, applies in Bertrand competition. Higher investment in period 1 lowers costs and raises output in period 2, presumptively mandating a higher *tax* (i.e., a lower value of s_2). Thus the future subsidy is a strategic *substitute* for the home firm's investment, encouraging the home firm to *under*-invest and so tending to justify a *subsidy* in period 1 to restore the efficient level of investment.

3.2 Learning By Doing

Learning by doing is similar to investment in capacity or R&D in the absence of policy (provided we continue to rule out inter-firm spillovers). However, when we consider its implications for policy, there are two important differences. First, since the firms compete directly in the first period, there is a profit-shifting motive for intervening in that period. Second, if competition in the first period is Bertrand, a firm which enjoys learning by doing faces different incentives from those in the investment case.

We have considered this case in Leahy and Neary (1994, 1998), drawing on Spence (1981) and Fudenberg and Tirole (1983), so our treatment can be brief. Because of learning by doing each firm's period-2 costs are inversely related to its period-1 output. Hence the second-period revenue function for the home firm becomes:

$$R^2(a_1,b_1,a_2,b_2) = \hat{R}^2(a_2,b_2) - C^2(q_1,q_2) , \qquad \textbf{19}$$

with R_1 positive if firms are Cournot competitors in the first period and negative if they are Bertrand competitors. Firms also compete directly in the first period, so the home firm's period-1 revenue function is now $R^1(a_1,b_1)$. Hence, no terms vanish from equations 11 and 11 for the optimal subsidies. Even in FCE there is both an intertemporal and an intratemporal profit-shifting motive for intervention in both periods. These terms are more complex than in static models but there is a presumption that they have the same implications for policy, mandating subsidies in both periods when competition is Cournot (provided outputs are strategic substitutes) and taxes in both periods when competition is Bertrand (provided prices

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[16] Since higher investment (an increase in a_1) lowers costs in period 2, the sign of this second strategic effect hinges on whether a reduction in costs mandates a higher subsidy in period 2. The study of how changes in costs affect the optimal subsidy was initiated by de Meza (1986) and Neary (1994), and in Leahy and Neary (1998) we derived a necessary and sufficient condition for this effect in Cournot competition. Appendix A.3 provides a simpler proof which applies to Bertrand as well as Cournot competition.

are strategic complements).

Consider next the inter-firm strategic effect. In Cournot competition this operates just as in the investment case. The home firm has an incentive to over-produce in period 1 and the government should tax its output in order to restrain this "top dog" behaviour. (Of course, the tax is relative to the profit-shifting benchmark, which is unambiguously positive provided outputs are strategic substitutes.) However, in Bertrand competition the parallel with the investment case breaks down, although the policy implication is the same. "Investment" in this case means a higher *price* in period 1 and, by lowering the rate of learning, this tends to raise the home firm's price in period 2 and so to *increase* the rival firm's profits. Hence, investment makes the home firm "soft" and, assuming that prices are strategic complements, it has an incentive to under-learn. The optimal policy response is thus to *exercise* this "fat cat" by reducing its current price, which in turn requires an output *subsidy*, as shown in the lower right-hand cell in Table 1.

Finally, the strategic effect vis-à-vis the government operates in the same way as in the investment case, presumptively mandating a tax in the Cournot case but a subsidy in the Bertrand case.

3.3 Natural Resources

The approach we have taken so far can be applied to the problem of optimal policy towards a resource-exporting firm competing against a single foreign rival. The simplest case is where each firm has a fixed stock of the resource, so in Cournot competition $a_1+a_2=A$ and $b_1+b_2=B$. In this case, both intertemporal and strategic considerations vanish. The model effectively collapses to a one-period one and the only motive for intervention is intratemporal profit shifting. The home firm's first-order condition is simply $R_1+s_1=r(R_2+s_2)$, which can be manipulated to give a variant of the Hotelling rule: the subsidy-inclusive marginal profitability of resource extraction falls at the rate of time preference.

Of more interest is the case where a higher rate of extraction in the first period raises the costs of extraction in the second.[17] This specification of the extraction technology is formally identical to learning by doing, except that the learning parameter is *negative*: future costs are *increasing* in current output. Hence higher output in the first period makes the home firm "soft" and so the case of Cournot competition falls into the upper-right-hand box of Table 1. Left to itself the home firm will under-produce in the first period, adopting a "lean and hungry look". Hence optimal intervention requires that it be *fattened* by a subsidy. Conversely, in Bertrand competition, a higher price in the first period makes the home firm "tough". Assuming prices are strategic complements, this leads it to adopt a "puppy dog" strategy in the sense that it *under-prices*; though the terminology is less appropriate here, since this implies that it *over-produces* and so the optimal policy (relative as always to the profit-shifting benchmark) is an output *tax*.

3.4 R&D Spillovers

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[17] Optimal policy in this case has been considered by Maskin and Newbery (1990) in a different context: they assumed that the resource was produced by a competitive industry and compared the optimal tariff schedules for a resource-importing country with and without government commitment.

In the examples considered so far, each firm's second-period costs depended only on its own first-period actions and not on its rival's. This simplicity vanishes if there are externalities between firms, as in the work of Spence (1984) and d'Aspremont and Jacquemin (1988). Moreover, the Fudenberg and Tirole taxonomy is no longer adequate, since from 14 there is an additional externality term in the expression for db_2/da_1. Some of the benefits of the home firm's R&D accrue to the foreign firm, reducing its costs and tending to raise its output. Because of this indirect effect, investment tends to make the home firm "soft", working against the direct effect which we have considered in Section 2.1 above.

To see which effect dominates, we must calculate explicitly the inter-firm strategic effect in the home firm's first-order condition 5. The home firm's revenue functions are now:

$$R^1(a_1) = -C^1(a_1), \qquad R^2(a_1,b_1,a_2,b_2) = \hat{R}^2(a_2,b_2) - C^2(a_1,b_1,q_2)$$
20

with corresponding expressions for the foreign firm. Differentiating the two firms' first-order conditions in period 2 is shown in Appendix A.4 to lead to:

$$p^2_{b_2}\frac{db_2}{da_1} = A(\overline{b} - b),$$
21

where:

$$\overline{b} \equiv \frac{R^{*2}_{b_2a_2}\,p^2_{a_2a_1}}{p^2_{a_2a_2}\,R^{*2}_{b_2b_1}} \quad \text{and} \quad b \equiv \frac{R^{*2}_{b_2a_1}}{R^{*2}_{b_2b_1}} = \frac{C^{*2}_{q_2a_1}}{C^{*2}_{q_2b_1}}.$$
22

As shown in Appendix A.4, second-order and stability conditions ensure that the term outside parentheses (*A*) is always positive. Hence, the spillover effect dominates, and investment makes the home firm "soft", if and only if the parameter we have labelled b is greater than that labelled . These two parameters are easily interpreted. b equals the ratio of "cross" to "own" effects of R&D on the foreign firm's marginal costs. It is natural to interpret this as a measure of the strength of spillovers, and to require it to lie between zero (the case of Section 2.1 with no spillovers) and one (the case where R&D is completely unappropriable). This leaves as the threshold value for b. The key feature of is that (as in the symmetric closed-economy model of Leahy and Neary (1997)), it is positive if and only if b_2 is a strategic substitute for a_2 (so that $R_{a2}<0$).[18] In the special case of homogeneous-product Cournot competition, linear demands and a symmetric quadratic specification of R&D, equals one half, the value found by d'Aspremont and Jacquemin (1988).

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[18] is the product of two terms. The second of these must be positive, since it equals the ratio of "own" effects of R&D on period-2 marginal profitability for the two firms. The denominator of the first term must be negative from the home firm's period-2 second-order condition. Hence the sign of depends solely on the sign of the numerator of the first term; i.e., on whether the foreign firm's period-2 action is a strategic substitute or a strategic complement for the home firms's.

Summarising, the total strategic effect leads the home firm to overinvest in R&D and so justifies an R&D tax for low spillovers (**b**<); whereas an R&D subsidy is justified for high spillovers (**b**>); and a subsidy is always justified when period-2 actions are strategic complements (since then **b**>0>). At first sight it may seem paradoxical that an R&D subsidy is more likely, the greater the extent of spillovers to the foreign firm, whose profits are of no concern to the home government. The source of the paradox lies in the strategic behaviour by the home firm, which leads it to under-invest in R&D when spillovers are high.[19]

2.5 Consumer Switching Costs

The final application we consider is to the case of switching costs in demand, as studied by Klemperer (1995) and To (1994). The period-2 revenue function can now be written as:

$$R^2(a_1,b_1,a_2,b_2) = \hat{R}^2[\boldsymbol{a}(a_1,b_1),a_2,b_2] . \qquad \textbf{23}$$

where **a** denotes the home firm's market share in period 1, equal to $q_1/(q_1+q)$. While the rationale for this specification differs greatly from that for R&D spillovers as considered in Section 2.4, they both imply the same form for the strategic effect. Thus equation 21 still holds in this case, the only difference being the form of the "spillover" parameter **b**:

$$\boldsymbol{b} \equiv \frac{R^{*2}_{b_2a_1}}{R^{*2}_{b_2b_1}} = \frac{\boldsymbol{a}^*_{a_1}}{\boldsymbol{a}^*_{b_1}} . \qquad \textbf{24}$$

b is now the ratio of "cross" to "own" effects of period-1 actions on the *foreign* firm's period-1 market share, $\boldsymbol{a}^*$. But since a_1 and b_1 have opposite effects on market shares (irrespective of whether competition in period 1 is in quantities or prices), **b** must be *negative*. (Though, as in Section 2.4, it is likely to be less than one in absolute value.) The implications for optimal policy are very similar to those in the previous sub-section. The only substantive difference is that the ambiguity in sign of the strategic component of the optimal subsidy now arises when actions are strategic complements, rather than when they are strategic substitutes. The threshold parameter is still positive if and only if period-2 actions are strategic substitutes; in that case we can be sure that **b**< and so the home firm "over-acts" in period 1, justifying a tax if competition is Cournot or a subsidy if competition is Bertrand. By contrast, if period-2 actions are strategic complements, then both **b** and are negative and the appropriate policy depends on their relative magnitudes.

4. Extensions

All the applications considered in Section 2 have remained within the framework of Section 1. This makes a number of restrictive assumptions, such as a passive foreign government, no home consumption and only one home firm. However, relaxing these introduces considerations

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[19] Further implications of strategic behaviour for policy towards R&D in the presence of both international and domestic spillovers are considered in Leahy and Neary (1999).

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which are not peculiar to an intertemporal framework and which are familiar from earlier work.[20] In the remainder of this section, we turn instead to three other, less familiar, directions in which the model of Section 1 needs to be extended.

4.1 Optimal Second-Best Intervention

So far, we have assumed that the home government can subsidise in both periods. However, governments often face constraints on their freedom to intervene. For example, the WTO (formerly the GATT) prohibits explicit export subsidies but does not constrain investment subsidies. Alternatively, it may be possible to evade the WTO prohibition on export subsidies (e.g., by providing export credits) but budgetary constraints may preclude direct assistance to investment. To understand such situations, it is desirable to extend the analysis of previous sections to the case where the government can vary only one instrument. We call this "second-best" intervention in contrast to the "first-best" intervention of previous sections (though of course it was first-best from a national, not a global, perspective, and truly first-best policy would require removing oligopolistic distortions).

If only one instrument is available, it matters crucially whether or not the government can commit to it in advance of any decisions by firms. Consider first the case where the government has the ability to commit. We can then adapt the methods of Section 1 to derive the optimal second-best value of the sole available instrument. Write the formulae for the first-best optimal subsidies (given by the right-hand sides of 11 and 11) as S_1 and S_2 respectively. Now we can immediately rewrite the change in welfare from 7 in terms of the *deviations* of the subsidy terms from their first-best values:

$$dW = -(S_{a_1} - \overline{S}_1) da_1 - (S_{a_2} - \overline{S}_2) da_2.$$

25

Now, if the period j subsidy is unalterable, the optimal value of the period i subsidy term is:

$$S^0_{a_i} = \overline{S}_i - (S_{a_j} - \overline{S}_j)\frac{da_j}{da_i}\bigg|_{s_j}, \quad i,j = 1,2; i \neq j.$$

26

Thus the optimal second-best subsidy term in period i equals the value given by the first-best formula S_i, less a correction factor which is the product of two terms: first, the deviation of the

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[20] See Brander (1995) for an overview and extensive references. Allowing for an active foreign government does not alter the formulae for the optimal home subsidies (except in SE), so there is a presumption that their signs are unchanged. However, if governments play a Nash game in subsidies, the actual equilibrium will be very different from the unilateral optimum, typically with lower welfare for both countries when firms play Cournot but higher when they play Bertrand. (See Leahy and Neary (1998).) Allowing for home consumption increases both the number of targets facing the government (consumer surplus in addition to corporate profits) and the number of instruments available to it (tariffs and production subsidies in addition to export subsidies). It may also change the model in other ways depending on whether the home and foreign markets are integrated or segmented. Finally, allowing for more than one domestic firm leads to pecuniary externalities between firms (unless they collude) which encourages export taxes for familiar terms of trade reasons. It also strengthens the case for subsidisation with R&D spillovers, unless firms engage in cooperative research joint ventures.

other subsidy term from its first-best value and, second, the extent to which, with s_j unalterable, a change in a_i affects a_j. The underlying rationale is similar to that in competitive models. (See for example, Neary (1995).) If the two home-firm actions a_i and a_j (which are the "targets" from the home government's perspective) are complements in the sense that da_j/da_i is positive, then the deviations of the two "instruments" S_{ai} and S_{aj} from their respective first-best optimal values should be opposite in sign.

To illustrate this result more concretely, note that the two equations in 26 define second-best loci which can be illustrated in subsidy space. Figures 3 and 4 show the loci implied by the investment model of Section 2.1.[21] In each figure the first-best optimum is denoted by point A; the contours are iso-welfare loci; the blank regions at the extremities of the figures correspond to subsidy values that are either too high or too low to sustain an interior equilibrium; and units are chosen such that welfare equals unity in free trade (i.e., at the $s_1=s_2=0$ origin).

Consider first the case of investment followed by Cournot competition (Figure 3). The two straight lines through A are the two second-best optimal subsidy loci defined by 26. Both are downward-sloping, reflecting the fact that the government's two targets, home investment and home output, are indeed complements in this case. However, the loci never stray into the south-west quadrant, so both second-best subsidies are positive when the other subsidy is constrained to equal zero. Spencer and Brander (1983) demonstrated this for the investment subsidy, though Figure 3 draws attention to a feature of their result which is inadequately appreciated. The motive for subsidising investment is only a second-best one, and is partly offset by the direct motive (the desire to counteract strategic over-investment) which works towards a tax. Hence the second-best investment subsidy is likely to be small in magnitude. This is true of the example illustrated in Figure 3: the second-best subsidy at point B is much lower in absolute value than the first-best tax at A and, in addition, leads to only slightly higher welfare than free trade.

Figure 4 shows the corresponding curves for the case of investment followed by Bertrand competition. As already noted in Section 2.1, the first-best package represented by point A exactly reverses that of the Cournot case in Figure 3: now the optimum involves taxing exports but subsidising investment. However, the second-best loci through A are both downward-sloping, reflecting two mutually offsetting differences from the Cournot case. First, the two targets, home investment and the home firm's price, are now *substitutes* rather than complements. (See, for example, equations 69 and 72 in Appendix A.6.) As a result, both subsidy *formulae* in equation 26 must lie on the same side of their respective first-best values. Offsetting this is the fact that, as always in Bertrand competition, S_{a2} is *negatively* related to the export subsidy, so the second-best optimal subsidy *rates* lie on opposite sides of their first-best values, just as in the Cournot case.

The other intriguing similarity between Figures 3 and 4 is that the optimal second-best investment subsidy is positive when the export subsidy is zero (at point B). This result was noted by Bagwell and Staiger (1994), whose findings were used by Brander (1995) as the basis for a conjecture that investment subsidies are a more robust recommendation (with respect to variations in assumptions about firm behaviour) than export subsidies. Figure 4 provides further evidence for this conjecture, drawing on the results of Maggi (1996). He

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[21] The underlying calculations and the parameter values chosen are given in the Appendix, Sections A.5 and A.6. Copies of the GAUSS programs used to generate Figures 3 to 6 are available on request.

considers a model such as that of Figure 4, with investment followed by price competition, concentrating on the sign of the optimal second-best *export* subsidy; i.e., on the location of point C. He focuses on the curvature of the marginal cost curve and shows that, if (as here) it is horizontal, the optimal second-best export subsidy is negative (as illustrated by point C in Figure 4). However, if the marginal cost curve is very steep, corresponding to a case where for a given level of investment the firm is constrained to produce at or near capacity output, then the optimal second-best output subsidy is *positive*. In terms of Figure 4, this implies that point C lies above the horizontal, and so point B must lie even further to the right of the origin, strengthening the case for a second-best optimal investment subsidy.[22]

The final case we must mention is where the government is constrained to offer an export subsidy only but cannot commit to it in advance of firms' first-period actions. It is obvious that, in that case, the "optimal" second-best policy will be given by the static formula 15. Of more interest is the fact that welfare may now be lower with an export subsidy than it would be in free trade. (Of course, this comparison assumes that it is feasible for the government to commit to free trade, which in itself implies some degree of government commitment.) The circumstances in which such a second-best "paradox" may arise have been considered by Karp and Perloff (1995), Grossman and Maggi (1997) and Neary and O'Sullivan (1997).

4.2 Divergences between Private and Social Costs

Returning to the case where both instruments are controllable, consider next the implications of a divergence between the social valuation of corporate profits and that of subsidy revenue foregone.[23] This implies that the welfare function depends directly on the subsidy rates for the first time:

$$W(a, b, s) = \boldsymbol{p}(a, b, s) - \boldsymbol{d}S(a, b, s), \quad \boldsymbol{d} \geq 1, \qquad 27$$

where the excess of the distributional parameter $\boldsymbol{d}$ over one measures the distributional preference for subsidy revenue over corporate profits, or alternatively, the deadweight losses from financing subsidies by distortionary taxation. Replacing $\boldsymbol{p}$ by $R+S$ as usual and calculating the government's first-order condition:

$$\frac{dW}{ds} = \frac{dR}{ds} - (\boldsymbol{d} - 1)\frac{dS}{ds} = 0. \qquad 28$$

The first term in 28 can be written in terms of the deviations of the optimal subsidy terms from

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[22] The same point can be made by considering equation 26, with $i=2$ and $j=1$. In the model with investment followed by Bertrand competition, $S_1=s_1>0$, $S_2=s_2q_{a2}>0$ (where q_{a2} is the own-price derivative of demand and is negative, so s_2 is negative), and $S_2=sq_{a2}$. If the marginal cost curve is very steep, da_1/da_2 (which is always negative) is very large in absolute value and so the second term in 26 dominates, implying that s is positive.

[23] As noted by Neary (1994), such a divergence may arise from a number of sources, including the deadweight loss of raising tax revenue to finance subsidies, pure distributional considerations, or the fact that the home firm is partly foreign-owned. A related type of divergence between private and social costs, which can be considered in similar fashion, is where changes in resource use by the home firm affect other distorted sectors. The implications for the pattern of subsidies across a number of oligopolistic sectors are considered by Dixit and Grossman (1986).

S, the vector of optimal subsidy formulae from Section 1 (i.e., when $d=1$):

$$\frac{dR}{ds} = (R_{a'} + R_{b'}B_a) A_s = -(S_a - \overline{S})' A_s \, , \tag{29}$$

where $a=A(s)$ is the reduced form equation giving the full effect of subsidies on the home firm's equilibrium actions.[24] The second term measures the response of subsidy revenue to changes in subsidy rates and is calculated similarly:

$$\frac{dS}{ds} = (S_{a'} + S_{b'}B_a) A_s + S_{s'} \, . \tag{30}$$

Substituting into the first-order condition 28 gives the optimal subsidies in this case:

$$S_{a'} = d^{-1} \overline{S}' - \tfrac{d-1}{d} (S_{b'}B_a + S_{s'}A_s^{-1}) \, . \tag{31}$$

This shows that S, the standard formula for the optimal subsidies, must be modified in two respects. First, the usual rent-shifting and strategic motives for intervention (summarised by S) are diluted, counting for less the higher is d. Second, there is an additional group of terms which reflect the deadweight loss of raising revenue. These terms are likely to contribute to a reduction in the optimal subsidies. (Consider the one-period case for example. If competition is Cournot, S_a is the optimal subsidy, while S_s and A_s are both positive; whereas if competition is Bertrand, then S_a is negatively related to the optimal subsidy, S_s is positive and A_s is negative.) However, there is no general presumption that this must be so in all cases. (Figure 5 below provides a counter-example in a simple case.) Nevertheless, we can show that *total* subsidy revenue unambiguously falls as d rises. Totally differentiate the subsidy function $S(a,b,s)$ with respect to d and substitute from the total differential of the government's first-order condition 28:

$$\frac{dS}{dd} = \left(\frac{d'S}{ds} \right) \frac{ds}{dd} = \left(\frac{d'S}{ds} \right) \left(\frac{d^2W}{ds^2} \right)^{-1} \frac{dS}{ds} \, . \tag{32}$$

This is a quadratic form in a matrix which must be negative definite from the government's second-order condition. Hence, S and d must be inversely related along the revenue-constrained iso-welfare locus.

   Figure 5 and 6, drawn in the same space and under the same assumptions as Figures 3 and 4 respectively, illustrate. Both diagrams repeat the iso-welfare contours from Figures 3 and 4, supplementing them with iso-profit contours and the revenue-constrained iso-welfare locus which passes through the first-best point A. A higher social opportunity cost of funds (i.e., a rise in d) implies a movement downwards from point A. However, as Figure 5 illustrates, this

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[24] This is the full effect (including the induced response of the foreign firm in both periods) as perceived by the home government, *not* by the home firm. Just as B_a is not equal to *db/da*, so also A^1 is not equal to *ds/da* (the response of the subsidies to a change in the home firm's actions as perceived by the firm).

implies in the Cournot case that the investment subsidy becomes *less* negative (though it never attains a positive value). The Bertrand case of Figure 6 is more intuitive. Here, both subsidies fall in algebraic value as **d** rises, though the investment subsidy never actually becomes negative.

4.3 Entry Promotion

The final case we consider is where the home firm's profits at the first-best optimum would be insufficient to induce it to enter the market. In this case the government's problem is to choose the optimal policy package to maximise welfare subject to a profit constraint. Forming the Lagrangian, this becomes:

$$\text{Max}_{s} \, \_ \, = \, W + \boldsymbol{l} \, (\, \boldsymbol{p} - \boldsymbol{p}_{m} \,) \, , \tag{33}$$

where $\boldsymbol{p}_{m}$ is the minimum required profit level. Assume the social cost of funds is equal to unity as in earlier sections, so $W=R$. Replacing $\boldsymbol{p}$ by $R+S$ as before, the first-order condition becomes:

$$(1 + \boldsymbol{l} \,) \frac{dR}{ds} + \boldsymbol{l} \, \frac{dS}{ds} \, = \, 0 \, , \quad \boldsymbol{l} \, \geq \, 0 \, . \tag{34}$$

But this is identical in form to 28, except that the coefficient of the deadweight-loss term is now positive rather than negative: $\boldsymbol{l} = \! (\boldsymbol{d}\! /\! 1)/\boldsymbol{d}$, or $\boldsymbol{d} = 1/(1 + \boldsymbol{l})$. Hence we can immediately, by analogy with 31, write down the optimal subsidy formula:

$$S_{a'} \, = \, (1 + \boldsymbol{l} \,) \overline{S'} \, + \, \boldsymbol{l} \, (\, S_{b'} B_{a} \, + \, S_{s'} A_{s}^{-1} \,) \, . \tag{35}$$

Now, the optimal subsidies are likely to, and total subsidy payments must, exceed their values in the unconstrained first-best optimum. Geometrically, the optimum lies along the same revenue-constrained iso-welfare loci in Figures 5 and 6 as before, but *above* rather than *below* the first-best point A.

5. Conclusion

In this paper we have developed a general approach to the design of optimal trade and industrial policy towards dynamic oligopolies. We have identified three distinct motives for intervention. First is the standard profit-shifting one of using the government's superior powers of commitment to place the home firm in the position it would adopt if it had a Stackelberg first-mover advantage over the foreign firm. An added complication is that, when firms compete directly in more than one period, there is scope for intertemporal as well as intratemporal profit-shifting. The second motive is the desire to counteract the strategic behaviour of the home firm vis-à-vis its rival. This type of strategic behaviour is summarised by Fudenberg and Tirole's taxonomy of business strategies and the optimal subsidy should contain a term which exactly offsets it. Finally, the third motive is the desire to counteract the home firm's strategic behaviour vis-à-vis the government's own future actions.

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In summary, the government should exploit its own strategic power and should counteract the home firm's. This means using its own powers of commitment both to shift profits (inter- as well as intratemporally) and to prevent the home firm from making socially wasteful commitments.

In addition to setting out the general principles which should guide intervention in a general dynamic oligopoly setting, we have examined their application in a number of special cases, including R&D (both with and without inter-firm spillovers), learning by doing, trade in natural resources and consumer switching costs. Our approach should prove fruitful in exploring issues of optimal intervention in the presence of many other types of intertemporal links. We have also shown how our general framework can be extended to allow for second-best, revenue-constrained and entry-promotion policies. In all cases the properties of particular models can be explained in terms of a few simple general principles, and we have also noted some useful analogies between seemingly different problems. For example, we have shown that, except for a sign change, there is a formal identity between models with learning by doing and natural resources; between R&D spillovers and consumer switching costs; and between revenue-constrained and entry-promotion policies.

Finally, what do our results imply for the practice of intervention targeted towards oligopolistic firms in open economies? It should be obvious that the paper confirms the general thrust of the literature on strategic trade and industrial policy, that no simple rules of thumb to guide policy are available. Not just the sign but even the magnitude of optimal intervention depends on subtle and perhaps unobservable aspects of firm behaviour. One general conclusion we can draw is some tentative support for a conjecture of Brander (1995), that the policy implications of strategic *industrial* policy may be more robust than those of strategic *trade* policy. Even in this case, as Sections 2.1 and 3.1 have shown, this policy rule is strictly a second-best one, in that it does not apply if exports as well as investment can be subsidised or taxed. Moreover, our simulations suggest that the potential gains from this limited form of intervention are likely to be small.

Yet the absence of any simple and universal rules for practical policy making does not detract from the intellectual significance of this branch of theory. In this respect, it may be helpful to draw an analogy between the theory of strategic trade policy and that great theoretical insight of the first decade of the twentieth century, Bickerdike's (1906) theory of optimal tariffs. For all sorts of practical reasons (of which the risk of foreign retaliation is the most obvious) the potential gains from manipulating the terms of trade are likely to be tiny, even for very large countries. Nevertheless, in trying to explain the welfare implications of actual interventions and the incentives facing governments in international negotiations, it is essential to understand the relationship between welfare and world prices and the potential if elusive gains from trying to exploit it. In exactly the same way, the potential role of a home government in taking on a Stackelberg leadership role which a home firm cannot take on for itself is a basic insight; even if the likelihood of realising the gains in practice is small. Given the enormous extent of intertemporal interdependence between firms, this suggests that the principles set out in this paper will continue to be relevant.

Appendix 1
Investment and Cournot Competition: Figure 1

We consider a linear-quadratic example where firms choose investment levels a_1 and b_1 in period 1 and outputs a_2 and b_2 in period 2. (This extends the example in Leahy and Neary (1996) to asymmetric firms.) Assuming no discounting ($r=1$), quadratic costs of investment, a linear demand function, homogeneous products and a linear relationship between investment and marginal cost, the home firm's profit functions are:

$$p^1 = -g\, a_1^2/2 + s_1 a_1 \qquad \text{and} \qquad p^2 = (p - c + s_2)a_2,$$
$$\text{where}: \quad p = a - b(a_2 + b_2) \qquad \text{and} \qquad c = c_0 - q\, a_1, \qquad \textbf{36}$$

with a similar specification (but different parameters g^*, c and q^*) for the foreign firm. The first-order conditions for output (defining the output reaction functions) are:

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} b\, a_2 \\ b\, b_2 \end{bmatrix} = \begin{bmatrix} a - c_0 + q\, a_1 + s_2 \\ a - c_0^* + q^* b_1 \end{bmatrix}. \qquad \textbf{37}$$

The first-order conditions for investment are:

$$mq\, a_2 = g\, a_1 - s_1 \qquad \text{and} \qquad mq^* b_2 = g^* b_1, \qquad \textbf{38}$$

where the parameter m reflects the strategic effect. When firms do not behave strategically, m equals unity and investment is at its efficient level in the absence of subsidies. Strategic behaviour adds extra terms in db_2/da_1 and da_2/db_1 (equal from 37 to $!q/3b$ and $!q^*/3b$ respectively) to the first-order conditions, raising the value of m to $4/3$. Hence both firms have an incentive to over-invest for strategic reasons. Solving for home investment in the absence of policy:

$$\hat{a}_1 = \frac{mh}{q} \frac{(2 - mh^*)(a - c_0) - (a - c_0^*)}{(2 - mh)(2 - mh^*) - 1}, \qquad \textbf{39}$$

where $h/q^2/bg$ and $h^*/(q^*)^2/bg^*$ measure the relative effectiveness of R&D for the home and foreign firm respectively. Perhaps surprisingly, a_1 need not be increasing in m for all parameter values. However, strategic behaviour leads to higher investment in equilibrium ($a_1[m=4/3] > a_1[m=1]$) when $h^* = 0$.

Consider next the implications of government intervention. Welfare equals (undiscounted) profits:

Totally differentiating and using the home firm's first-order conditions to simplify:

To eliminate db_2 we need to solve for the slope of the generalised foreign reaction function (as in 9) using both the foreign first-order conditions from 37 and 38. Solving these gives db_2/db_2 equal to $!1/(2!mh^*)$, so substituting into 41 gives:

$$\frac{dW}{da_1} = -g\, a_1^2/2 + (p - c)\frac{da_2}{} \qquad \textbf{40}$$

$$\textbf{41}$$

$$dW \; = \; - \, (\, s_1 - \bar{s}_1 \,) \, da_1 - (\, s_2 - \bar{s}_2 \,) \, da_2 \, ,$$

$$\text{where}: \; \bar{s}_1 = - (m-1) q \, a_2 \quad \text{and} \quad \bar{s}_2 = \frac{1}{2 - mh^*} \, b \, a_2 \, . \tag{42}$$

The formulae for the first-best optimal subsidies (specialising 11 and 11) confirm the results of Spencer and Brander (1983): investment is taxed when firms behave strategically but not otherwise; while exports are always subsidised, the more so if the foreign firm behaves strategically. Finally, we can solve for the level of home investment:

$$\bar{a}_1 \; = \; \frac{h}{q} \; \frac{(2 - mh^*)(a - c_0) - (a - c_0^*)}{(2 - h)(2 - mh^*) - 2} \, , \tag{43}$$

which is independent of m when $h^*=0$, in accordance with Proposition 4. Comparing 39, investment is higher at the social optimum ($a_1 > a_1$) for either value of m

Figure 1 illustrates the special case where there is no foreign investment (so $h^*=q^*=0$). The lower panel illustrates the home firm's first-order condition for investment from 38, with $s_1=0$ and m equal to either unity (along OK) or $4/3$ (along OKN). The upper panel illustrates the reaction functions from 37, with the appropriate values of a_1 and s_2 substituted to obtain the three home curves.

We assume the same specification of costs as in Appendix A.1, and a linear, differentiated-products specification of demand (with a_2 and b_2 now denoting prices and q and q^* denoting home and foreign outputs):

$$p^1 = -g\, a_1^2/2 + s_1 a_1 \qquad \text{and} \qquad p^2 = (a_2 - c + s_2) q,$$

$$\text{where}: \quad q = a - b(a_2 - b_2) \qquad \text{and} \qquad q^* = a - b(b_2 - a_2) \qquad 44$$

The first-order conditions for price (defining the price reaction functions) are:

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} b\, a_2 \\ b\, b_2 \end{bmatrix} = \begin{bmatrix} a + b c_0 - bq\, a_1 - b\, s_2 \\ a + b c_0^* - b q^* b_1 \end{bmatrix}. \qquad 45$$

The first-order conditions for investment are:

$$mqq = g\, a_1 - s_1 \qquad \text{and} \qquad mq^* q^* = g^* b_1, \qquad 46$$

where the parameter m reflects the strategic effect. When firms do not behave strategically, m equals unity and investment is at its efficient level. Strategic behaviour adds extra terms in db_2/da_1 and da_2/db_1 (equal from 37 to $!q/3$ and $!q^*/3$ respectively) to the first-order conditions, reducing the value of m to $2/3$. Hence both firms have an incentive to under-invest for strategic reasons. Solving for home investment in the absence of policy:

where $h/bq^2/g$ and $h^*/b(q^*)^2/g^*$ measure the relative effectiveness of R&D for the home and foreign firm respectively. Strategic behaviour leads to lower investment in equilibrium (a $_1[m=2/3] < a_1[m=1]$) for most admissible parameter values (at least for $h \# 1$ and $c_0 \# c$).

Turning to policy, the welfare function is the same as in Section A.1 and its total differential (using the home firm's first-order conditions to simplify) is:

Once again, we need to eliminate db_2 by solving for the slope of the generalised foreign reaction function (as in b) using both the foreign first-order conditions from 45 and 46.

Solving these gives B_{22} equal to $(1!mh^*)/(2!mh^*)$, so substituting into 48 gives:

where:

As noted in the text, the formulae for the first-best optimal subsidies exactly reverse the results of Spencer and Brander (1983): investment is subsidised when firms behave strategically but not otherwise; while exports are always taxed, though by less if the foreign firm behaves strategically. Finally, we can solve for the level of home investment:

Once again, this is independent of m when $h^* = 0$, in accordance with Proposition 4. Comparing 47, investment is lower at the social optimum when firms do not behave strategically ($m = 1$) but higher when they do ($m = 2/3$).

Figure 2 illustrates the special case where there is no foreign investment (so $h^* = q^* = 0$). The lower panel illustrates the home firm's first-order condition for investment from 46, (translated into (a_1, a_2) space by using the foreign firm's first-order condition to eliminate b_2)

with $s_1=0$ and $\boldsymbol{m}$ equal to either unity (along KK) or *2/3* (along KNKN). The upper panel illustrates the price reaction functions from 45, with the appropriate values of a_1 and s_2 substituted to obtain the three home curves.

Appendix 3
Derivation of ds_2/da_1

To calculate ds_2/da_1 in SE, we need to solve jointly all three first-order conditions in period 2: one each for the home firm, the foreign firm and the government. We must also consider the government's problem in detail, since its second-order condition implies a useful restriction. We simplify by ignoring the dependence of R^{*2} and S^2 on a_1: thus our results apply only to the cases of investment, learning by doing and natural resources of Sections 2.1 to 2.3. In addition, for convenience we suppress b_1 from all the functions, since we need consider only the home firm's behaviour.

Since the foreign firm is effectively passive in period 2, we may solve its first-order condition for its reaction function:

$$b_2 = \mathbf{y}(a_2).$$
<div align="right">52</div>

Taking this into account but treating a_1 as predetermined, the government's problem is to maximise period-2 welfare:

$$\underset{s_2}{\operatorname{Max}} W^2 = R^2[a_1, a_2, \mathbf{y}(a_2)].$$
<div align="right">53</div>

This leads to the first-order condition (which in turn leads to 15 in the text):

$$\frac{dW^2}{ds_2} = [R_{a_2}^2\{a_1, a_2, \mathbf{y}(a_2)\} + R_{b_2}^2\{a_1, a_2, \mathbf{y}(a_2)\} \cdot \mathbf{y}_{a_2}(a_2)] \frac{da_2}{ds_2}.$$
<div align="right">54</div>

Totally differentiating 54 and evaluating at the optimum gives the second-order condition for the government's maximisation problem:

$$\left. \frac{d^2 W^2}{ds_2^2} \right|_{s_2 = s_2^s} = [R_{a_2a_2}^2 + R_{a_2b_2}^2 \mathbf{y}_{a_2} + (R_{b_2a_2}^2 + R_{b_2b_2}^2 \mathbf{y}_{a_2})\mathbf{y}_{a_2} + R_{b_2}^2 \mathbf{y}_{a_2a_2}] \left(\frac{da_2}{ds_2}\right)^2.$$
<div align="right">55</div>

Hence, irrespective of the sign of da_2/ds_2, the second-order condition implies that the term in square brackets must be negative. Call this term E.

Now, totally differentiate the government's first-order condition from 54:

Finally, totally differentiate the home firm's first-order condition $p_2=0$ to obtain:

$$da_2 = -E^{-1}(\bar{R}_{a_2a_1}^2 + \bar{R}_{b_2a_1}^2 \bar{\mathbf{y}}_{a_2}) da_1.$$
<div align="right">56</div>

$$(\mathbf{p}_{a_2a_2}^2 + \mathbf{p}_{a_2b_2}^2 \mathbf{y}_{a_2}) da_2 + R_{a_2a_1}^2 da_1 + S_{a_2s_2}^2 ds_2 = 0,$$
<div align="right">57</div>

and use 56 to eliminate da_2:

This is the required expression for ds_2/da_1. The only general restrictions which apply are that E is negative from the government's second-order condition, as already noted, and that the final bracketed expression on the right-hand side, $p_{2a2}+p_{2b2}y_{a2}$, is negative from the stability of the period-2 game. We therefore turn to consider some special cases.

In Cournot competition a number of terms simplify: $S_{2s2}=1$, $R_{2a1}=!C_1>0$ and $S_{2a2}=S_{2b2}=R_{2a1}=0$. If in addition we assume that products are homogeneous, standard properties of the revenue function are that $R_{2a2}=p(1+a_2r_2)$ and $R_{2b2}=pa_2r_2$, where a_2 is the market share of the home firm, p is the slope and r_2 $(=(a_2+b_2)p''/p)$ is the degree of concavity of the demand function. Hence 58 simplifies to:

$$(-E)\frac{ds_2}{da_1} \;=\; p_2 y_{a_2} [1 + a_2 r_2 (1+y_{a_2}) + a_2 y_{a2a2}/y_{a_2}](-C_{a_1}^2).$$

$$\textbf{59}$$

$$(-E)S_{a2s2}^2\,\frac{ds_2}{da_1} \;=\; [(R_{b2a2}^2+R_{b2b2}^2 y_{a_2})y_{a_2}+R_{b2}^2 y_{a2a2}-(S_{a2a2}^2+S_{a2b2}^2 y_{a_2})]$$

$$-(p_{a2a2}^2+p_{a2b2}^2 y_{a_2})y_{a_2}R_{b2a1}^2.$$

$$\textbf{58}$$

Since p, y_2 and C_1 are all negative, ds_2/da_1 is positive if and only if the expression in square brackets is positive, confirming Proposition 3 in Leahy and Neary (1998). With linear demands, $r_2=y_{a2a2}=0$, and the expression reduces to unity. More generally, there is still a presumption that it is positive provided neither the demand function not the foreign reaction function is "too" convex.

In Bertrand competition, no terms disappear but we have a number of restrictions: $y_{a2}>0$, assuming prices are strategic complements; $S_{2s2}=q_2<0$, $R_{2a1}=!C_1q_2<0$ and $R_{2a1}=!C_1q_2<0$ (recalling that a_2 and b_2 are home and foreign prices respectively). With linear demands, 58 reduces to:

$$(-E)\frac{ds_2}{da_1} \;=\; -q_{b_2}^2 y_{a_2}[1+\frac{q_{b_2}^2}{q_{a_2}^2}y_{a_2}](-C_{a_1}^2),$$

$$\textbf{60}$$

where the slope of the foreign reaction function is: $y_{a2}=!q/2q>0$. Assuming that own-price effects dominate cross-price effects, the right-hand side of 60 must be negative and so higher investment reduces the period-2 subsidy (or, rather, increases the period-2 tax). This continues to be the general presumption, as long as the second derivatives of the demand and foreign reaction functions do not dominate.

Appendix 4
Strategic Effects with R&D Spillovers or Consumer Switching Costs

To derive the inter-firm strategic effect given in 21, we need to calculate the effects of period-1 actions on period-2 variables. Differentiating totally the two period-2 first-order conditions, $p_2=0$ and $R=0$, yields:

$$\begin{bmatrix} p^2_{a_2a_2} & p^2_{a_2b_2} \\ R^{*2}_{b_2a_2} & R^{*2}_{b_2b_2} \end{bmatrix} \begin{bmatrix} da_2 \\ db_2 \end{bmatrix} = \begin{bmatrix} {}^- p^2_{a_2a_1}\, da_1 \;-\; p^2_{a_2b_1}\, db_1 \\ - R^{*2}_{b_2a_1}\, da_1 \;-\; R^{*2}_{b_2b_1}\, db_1 \end{bmatrix}. \tag{61}$$

Solving for db_2/da_1 gives:

$$\Delta\, \frac{db_2}{da_1} \;=\; -\, p^2_{a_2a_2}\, R^{*2}_{b_2a_1} \;+\; p^2_{a_2a_1}\, R^{*2}_{b_2a_2}, \tag{62}$$

with D denoting the determinant of the coefficient matrix in 61. This leads to equation 21 in the text, where the term A is defined as:

$$A \equiv p^2_{b_2}\, \frac{p^2_{a_2a_2}\, R^*_{b_2b_1}}{\Delta}. \tag{63}$$

The determinant is positive from stability of the period-2 game and the term p_{2a2} is negative from the home firm's second-order condition. As for the other two terms in A, they always differ in sign. If period-2 competition is Cournot, then p_2 is negative (a rise in foreign output reduces home profits) and R_{b1} is positive (a rise in foreign investment tends to raise foreign output). Both signs are reversed in Bertrand competition. The net effect is therefore that A must be positive.

Appendix 5
Second-Best Investment and Output Subsidies in Cournot Competition: Figure 3

Returning to the model of Section A.1, we wish to solve 42 for the optimal second-best subsidy loci given by 26 in the text. We concentrate on the GCE case (setting $m=4/3$); the FCE case is qualitatively similar. First, to derive the second-best optimal export subsidy, we use the home first-order condition for investment from 38, with ds_1 set equal to zero, to eliminate da_1. This gives:

$$\frac{da_1}{da_2}\bigg|_{s_1} = \frac{4q}{3g}.$$

$$64$$

Substituting into 42 and replacing s_1 and s_2 by the first-best optimal subsidy formulae:

$$s_2^0 = \bar{s}_2 - \frac{4q}{3g}(s_1 - \bar{s}_1) = -\frac{4q}{3g}s_1 + \frac{27 - 8h(3 - 2h^*)}{18(3 - 2h^*)}b\,a_2$$

$$65$$

To final step is to express a_2 in terms of s_1 and s_2 and substitute into 65. An explicit expression for a_2 may be found by using 38 to eliminate a_1 and b_1 from 37 and solving:

$$\Delta b\,a_2 = 2(1 - \tfrac{2}{3}h^*)\Big[a - c_0 + \tfrac{q}{g}s_{1+s_2}\Big] - (a - c_0^*),$$

$$\text{where: } \Delta \equiv 4(1-\tfrac{2}{3}h)(1-\tfrac{2}{3}h^*) - 1.$$

$$66$$

Combining 65 and 66, it can be checked that s is declining in s_1 for admissible (i.e., low) values of h and h^*.

Next, to calculate the second-best optimal investment subsidy, combine both foreign first-order conditions with the home first-order condition for output from 37, with ds_2 set equal to zero, to obtain:

$$\frac{da_2}{da_1}\bigg|_{s_2} = \frac{2(3 - 2h^*)}{9 - 8h^*}\frac{q}{b}$$

$$67$$

Using this to eliminate da_2 from 42 as before gives the second-best optimal investment subsidy:

$$s_1^0 = \bar{s}_1 - \frac{2(3 - 2h^*)}{9 - 8h^*}\frac{q}{b}(s_2 - \bar{s}_2) = -\frac{2(3 - 2h^*)}{9 - 8h^*}\frac{q}{b}s_2 + \frac{8h^*}{9(9 - 8h^*)}q\,a_2$$

$$68$$

Once again, we need to eliminate a_2 using 66, which shows that s is also declining in s_2 for admissible (i.e., low) values of h and h^*.

Figure 3 illustrates the symmetric case, ($h=h^*$ and $c_0=c$), with the normalisation

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$b=q=a!c_0=1$ and a common value for h of 0.2. No interior solution is possible in the blank regions, since either subsidies are sufficiently high to drive foreign output and investment to zero or taxes are sufficiently high to drive one or both of home output and investment to zero.

Appendix 6
Second-Best Investment and Output Subsidies in Bertrand Competition:
Figure 4

The steps in this derivation are identical to those in the Cournot case of Section A.5. Returning to the model of Section A.2, we wish to solve 49 for the optimal second-best subsidy loci given by 26 in the text, confining attention to the GCE case (where $m=2/3$). First, to derive the second-best optimal export subsidy, we use the home first-order condition for investment from 46, with ds_1 set equal to zero, to eliminate da_1. This gives:

$$\left.\frac{da_1}{da_2}\right|_{s_1} = -\frac{1}{3-h^*}\frac{bq}{g} \qquad \textbf{69}$$

Substituting into 49 and replacing s_1 and s_2 by the first-best optimal subsidy formulae:

$$s_2^0 = \bar{s}_2 - \frac{2q}{3g}\left(s_1 - \bar{s}_1\right) = -\frac{2q}{3g}s_1 - \left(9-2h-6h^*\right)\frac{q}{9b} \qquad \textbf{70}$$

As before, we need to express q in terms of s_1 and s_2. An explicit expression for q may be found by using 45 to eliminate a_1 and b_1 from 46, solving for a_2 and b_2, and then substituting into the demand function in 44:

$$\Delta q = \left(3 - \tfrac{4}{3}h^*\right)a - b(c_0 - c_0^*) + b\left(\tfrac{q}{g}\,s_1 + s_2\right),$$

$$\text{where: } \Delta \equiv 3 - \tfrac{2}{3}(h+h^*). \qquad \textbf{71}$$

Combining this with 70, it can be checked that s is declining in s_1 for admissible (i.e., low) values of h and h^*.

Next, to calculate the second-best optimal investment subsidy, combine both foreign first-order conditions with the home first-order condition for price from 45, with ds_2 set equal to zero, to obtain:

$$\left.\frac{da_2}{da_1}\right|_{s_2} = -\frac{2(3-h^*)}{9-2h^*}\,q \qquad \textbf{72}$$

Using this to eliminate da_2 from 49 as before gives the second-best optimal investment subsidy:

$$s_1^0 = \bar{s}_1 - \frac{3bq}{9-2h^*}\left(s_2 - \bar{s}_2\right) = -\frac{3bq}{9-2h^*}s_2 + \frac{2(9-4h^*)}{3(9-2h^*)}qq \qquad \textbf{73}$$

Once again, we need to eliminate q using 71, which shows that s is also declining in s_2 for admissible (i.e., low) values of h and h^*.

Figure 4 illustrates the symmetric case, ($h=h^*$ and $c_0=c$), with the normalisation $b=q=a=1$ and a common value for h of 0.4.

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