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Models and reality: the curious case of the absent abstention

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Models and Reality: The Curious Case of the Absent Abstention[1]

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Abstract

We discuss two inter-related puzzling features of the literature on a priori voting power. First, the mathematical model used in virtually all this literature does not recognize abstention as an option distinct from both a 'yes' and a 'no' vote. Second, real-life decision rules of voting bodies—in particular the US legislature and the UN Security Council—are misrepresented as though they did not allow abstention as a *tertium quid*. We suggest that these misrepresentations may be examples of what philosophers of science call 'theory-laden observation'.

Models and Reality: The Curious Case of the Absent Abstention

1 Introduction

In this paper we address some methodological issues that arose in connection with our technical work—mainly Felsenthal and Machover (1997), but see also Felsenthal and Machover (1995, 1996, 1998) and Felsenthal, Machover and Zwicker (1998).

Virtually all the work published so far on a priori voting power has used, explicitly or implicitly, one and the same type of mathematical structure to model decision rules of voting bodies that make yes/no decisions. We shall refer to a structure of this type as a *simple voting game* (briefly, *SVG*). Some authors consider arbitrary SVGs, whereas others confine themselves to an important subclass—that of *weighted voting games* (briefly, *WVGs*). We shall assume that the reader is familiar with the definitions of these concepts, as codified by Shapley (1962) and reproduced by other authors (for example, Straffin, 1982, Felsenthal and Machover, 1995). We shall also assume familiarity with the definitions of the main indices used for measuring a priori voting power: the Shapley–Shubik (S-S) index, denoted by 'ϕ' (see Shapley and Shubik, 1954); and the Banzhaf (Bz) index (see Banzhaf, 1965). The relative version of the Bz index (normalized so that the sum of its values for all voters in any SVG add up to 1) is denoted by 'β', and the so-called absolute Bz index is denoted by 'β'' (see Dubey and Shapley, 1979; Straffin, 1982; or Felsenthal and Machover, 1995).[1]

An obvious fact about the SVG set-up is that it is strictly *binary*: it assumes that in each division[2] a voter has just two options: voting 'yes' or 'no'. On the other hand, many real-life decision rules are *ternary* in the sense that they allow abstention[3] as a *tertium quid*, which may have different effects

[1] As a matter of fact, the Bz index is essentially a re-invention of a measure of voting power that had been proposed by Lionel Penrose (1946). In the present notation, the measure proposed by Penrose was $\beta'/2$.

[2] We borrow the term 'division' from English parliamentary parlance, to denote the *collective* act of a decision-making body, whereby each individual member casts a vote. Somewhat surprisingly, writers on voting power have not made this necessary terminological distinction between the collective and the individual acts, and refer to both of them as 'voting'.

[3] Unless the contrary is indicated, we use the term 'abstention' in its wide sense, in-

from both a 'yes' and a 'no' vote, and so cannot be assimilated to either.[4]

This raises several interrelated questions, which we shall address in the following three sections.

In Section 2 we shall see how writers on voting power deal *theoretically* with the issue of abstention, and consider whether their treatment is adequate. This will lead us into a discussion of a sadly neglected distinction, first mooted by Coleman (1971), between two alternative notions underlying the formal measurement of voting power.

In Section 3 we shall see what adjustments, if any, scholars make when *applying* the binary model to real-life situations that are essentially ternary, and how they report the facts about such ternary rules. Here we shall conduct two brief case studies of a phenomenon familiar to philosophers of science, who refer to it as 'theory-laden observation'.

In Section 4 we shall consider whether an adequate ternary theoretical model can be set up; and if so, whether it yields significantly different results concerning the measurement of voting power.

2 Theoretical discussion of abstention

Theoretical discussion of abstention is conspicuous by its almost total absence in the literature on voting power.

An exception—which, in a sense, proves this rule—is Morriss's book (1987, Chapters 21–24), which lies outside the mainstream publications on voting power. One of the virtues of that idiosyncratic work is that it takes abstention seriously, and makes no attempt to brush it under the carpet. However, Morriss does not propose an index for measuring a priori voting power (which he calls 'ability') in the presence of abstentions, nor does he define a ternary analogue of the binary SVG structure. He does outline a method of measuring a posteriori voting power (which he calls 'ableness'). We shall not enter here into an assessment of the adequacy of his outline, because in this paper we are concerned with a priori power.

As far as the mainstream literature is concerned, the only positive treatment of abstention we have come across is in Fishburn's book (1973, pp. 53–55). Fishburn considers what might be called *self-dual ternary weighted voting games*: voters are assigned non-negative weights and are asked to express a preference or indifference between two outcomes, x and y (which

cluding an explicit declaration 'I abstain', non-participation in the division, or absence.

[4]Note that even under a ternary rule, the outcome of a division is still a dichotomy: the bill in question must either be adopted or rejected, *tertium non datur*.

may, in particular, be answers 'no' and 'yes' to a given question; in this case indifference amounts to abstention). The winning outcome is that whose supporters have greater total weight than the supporters of the alternative outcome. For such decision rules Fishburn defines a straightforward generalization of the Bz index. These ternary voting games are a very restricted class; for example, it is easy to see that the decision rule of the UN Security Council (UNSC) cannot possibly be cast in this mould.[5] Also, he makes no attempt to generalize the S-S index to his ternary weighted games. Despite its limitations, Fishburn's (1973) brief treatment is a very significant positive step. But as far as we know it was not developed further in the literature published in the following 20 years.[6]

How do other writers on voting power justify their practice of confining themselves to binary theoretical models, which do not admit abstention?

Banzhaf dismisses the issue of abstention with a brief remark encaved in a footnote, (cf. Banzhaf 1965, fn. 34):

> This analysis has also assumed that all legislators are voting because this is the most effective way for each legislator to exercise his power. Naturally, some may choose to exercise their power in a less effective manner by abstaining or by being absent from the legislative chamber.

Banzhaf's argument for disregarding abstentions seems to us inadequate, as we shall explain below. But at least he does not ignore the whole issue, as do other writers.

In the published literature, as far as it is known to us, we have not found any other attempt to provide theoretical justifications for disregarding abstentions. But when presenting an earlier version of this paper at an interdisciplinary seminar, some of the game theorists in the audience reacted rather heatedly with a somewhat more elaborate form of Banzhaf's argument, which may be paraphrased as follows.

> The study of voting power belongs to game theory; more specifically, it is a branch of the theory of n-person games. Game theory is a theory of rational behaviour. Abstaining voters are not behaving rationally, because they are not using their powers to the

[5]Note, however, that in one respect Fishburn's model is *too* general from the viewpoint of yes/no decision-making: the model allows unresolved ties, which occur if the supporters of outcomes x and y have equal total weights. But in voting on a resolution or a bill, the outcome must be either 'yes' or 'no'; see footnote 4. The normal practice is that in case of a tie the status quo remains, so that a tie is resolved as a 'no'.

[6]See however Addendum at the end of Section 3 below.

> full. Therefore such behaviour ought to be disregarded by the theory.

This argument presupposes a particular view of voting behaviour. According to this view, advocated by Shapley, the 'worth' assigned to a coalition by the characteristic function of an SVG—1 to a winning coalition and 0 to a losing coalition—is not just a formal label. Rather, according to this view the worth of a coalition S represents the total payoff that the members of S earn when S is the set of 'yes' voters in a division.

What is this total payoff? Shapley's answer is quite explicit: 'the acquisition of power is the payoff' (see Abstract in Shapley 1962, p. 59).

The idea is that in winning a division, the winning coalition captures a fixed purse—the prize of power—which it then proceeds to divide among its members. The formation of the winning coalition as well as the distribution of the spoils among its members are consequent upon a process of bargaining. The motivation of voting behaviour that this view assumes has been called 'office seeking' by political scientists. It is this view of voting behaviour that underlies the S-S index and provides the justification for regarding it as a measure of voting power: the voting power of a voter is conceptualized as his or her expected or estimated share in the loot of power.

Now, it is true that from this office-seeking perspective on voting, abstention may be regarded as irrational: if by voting 'yes' you can get a share of the spoils as a member of a winning coalition that acquires power, then vote 'yes'; otherwise vote 'no'. You'll never get a prize for sitting on the fence.

But there is an alternative possible motivation of voting behaviour, which political scientists have called 'policy seeking'.[7] In his perspicacious critique of the S-S index, Coleman (1971, p. 272) points out that the latter motivation is the more usual,

> ... for the usual problem is not one in which there is a division of the spoils among the winners, but rather the problem of controlling the action of the collectivity. The action is ordinarily one that carries its own consequences or distribution of utilities, and these cannot be varied at will, i.e. cannot be split up among those who constitute the winning coalition. Instead, the typical question is ... the passage of a bill, a resolution, or a measure committing the collectivity to an action.

[7]For a discussion of these alternative motivations in a political context, see Laver and Schofield (1990, esp. Ch. 3).

Indeed, this seems a realistic account of voting in, say, the UNSC. Incidentally, the UNSC is one of the two examples given by Shapley (1962, p. 59) of a 'body in which the acquisition of power is the payoff'; but it is not at all clear how passing a resolution in this body amounts to acquisition of power by those voting 'yes'.

Of course, as Coleman admits, there are some circumstances where voting behaviour is motivated wholly or partly by office seeking. But the more usual cases are those where policy seeking is the predominant motivation. In these cases the outcome of a division of a decision-making body—the passage of a bill or its defeat—creates a *public good*, to which each voter may attach a utility value. This value (rather than the voter's share in some fixed purse) is the payoff that the voter ought to maximize.

It is this view of voting that underlies the absolute Bz index and its variants, proposed by Penrose and Coleman, and justifies its use as a measure of voting power: here a voter's power is conceptualized as the degree to which (or the probability that) he or she is able to affect the outcome of a division.[8]

Now, from a policy-seeking perspective on voting, the argument for disregarding abstention in theory loses most of its force. There may be several reasons why a voter would wish to abstain on a given bill. One reason can be the wish to use abstention as a way of making a public statement. The voter expects to derive some benefit not only from abstaining, but *being seen to abstain*. Such abstention should perhaps be disregarded by the theory of voting power, because it depends on the propaganda advantage of abstention itself, as a kind of side payment. Note that abstention for propaganda cannot operate if voting is secret. But in our view there are also other reasons for abstaining, which operate even when voting is secret. A voter may be indifferent to the bill, because his or her interests are not affected by it in any way. (This is a reasonable motive for abstention by absence, particularly if participation in the division involves some cost.) Or the arguments for and against the bill—the estimates of the payoff to the given voter in case the bill is adopted or rejected—may be so finely balanced that the voter is unable to decide one way or the other. Is it so irrational to abstain for these reasons? It is a bizarre kind of rationality that would require you to cast a 'yes' or 'no' vote even when you couldn't care less, or when you were not sure whether passage of the bill would serve your interests better than its defeat!

The study of voting power is a branch of social-choice theory. In other

[8]For an elaboration of Coleman's distinction between the two conceptions of voting behaviour as underpinning two kinds of power indices, see Felsenthal, Machover and Zwicker (1998); and Felsenthal and Machover (1998, Comment 2.2.2, Section 3.1 and Section 6.1).

branches of the theory—for example, in the study of social choice functions—it is quite normal to admit individual preference rankings that are not totally ordered but rank one or more outcomes (or candidates) as coequal. It is not that questions of individual rationality are ignored: for example, it is often argued and widely accepted that non-transitive individual preference rankings ought to be disallowed, precisely on the ground that they are not rational. But to the best of our knowledge there are not many social-choice theorists who would condemn as irrational an individual voter who does not wish or is unable to choose between Tweedledum and Tweedledee, or even between the Walrus and the Carpenter.[9] Why should the theory of voting power be different in this respect?

3 Theory-laden observation

In Section 2 we argued that abstention can be rational and that it should be allowed as a legitimate option in the theory of voting power. This, of course, is a matter of opinion, on which some readers may well disagree with us. We now turn to matters of fact, on which presumably there ought to be no controversy: the decision rules actually operated by certain real-life voting bodies, and the way these rules are reported in the literature on a priori voting power.

In this literature, four real-life cases are used as stock examples to illustrate the application of the theory to the real world. These are the US legislature (consisting of the two Houses of Congress and the veto-wielding President), the UN Security Council (UNSC), the mechanism (enacted in 1982) for amending the Canadian constitution, and the so-called qualified majority rule applied by the European Union's Council of Ministers to matters of a certain type. The last two examples need not concern us here, as they do indeed exclude abstention as a distinct option and can therefore be

[9]Social choice theorists explicitly recognize that voters may be unable to choose between two or more alternatives, and may prefer to abstain rather than select arbitrarily one of the alternatives among which they are indifferent. Thus, for example, Brams and Fishburn (1983, pp. 3–4) state that one of the advantages of *approval voting*—in which each voter can cast one vote for each candidate of whom he or she approves, and the candidate who obtains the largest number of votes is elected—is that voters 'who have no strong preference for one candidate, ... can express this fact by voting for all candidates they find acceptable ... [and thus voters] who cannot decide which of several candidates best reflects their views, would not be on the horns of a dilemma. By not being forced to make a single—perhaps arbitrary—choice, they would feel that the election system allows them to be more honest. We believe this would make voting more meaningful and encourage greater participation in elections.'

treated as SVGs, at least as far as abstention is concerned.[10]

Matters are quite different in the US legislature and the UNSC. The rules of these bodies do in fact treat abstention or absence as a *tertium quid*.

First, let us consider the US legislature. Article 1, Section 5(1) of the US Constitution stipulates that business in each of the two Houses of Congress can only take place if a (simple) majority of its members are present. Beyond this, the Constitution leaves it to the two Houses to fix their own rules of decision on most matters. The practice is that in each House an ordinary bill (as distinct from a decision to override a presidential veto) is deemed to pass if the necessary quorum is present and a simple majority of the members *participating in the division* vote 'yes'.[11] (The Vice President, in his role as President of the Senate, has only a casting vote, which he can use to break ties.)

The US Constitution explicitly refers to members *present* in only two instances, both concerning the Senate. Thus Article 1, Section 3(6) stipulates that in cases of impeachment the Senate's decision to convict requires the assent of at least two-thirds of the members *present*. So a President could, in theory, be convicted by the assent of just over one-third of all members, against the 'no' of just under one-sixth, with just under one-half of the members absent. Similarly, Article 2, Section 2(2), stipulates that the President shall have power, by and with the advice and consent of the Senate, to make treaties, provided two-thirds of the Senators *present* concur.

In case of a presidential veto, Article 1, Section 7(2) of the Constitution

[10] Amendment of the Canadian constitution requires an assent of at least two-thirds of the provinces, inhabited by at least one-half of the total population; so here abstention counts as a 'no'.

In matters that require assent of a qualified majority of the European Union's Council of Ministers, as defined by Article 148(2) of the Single European Act, abstention counts as a 'no' vote because a fixed number of votes must be obtained in order for a resolution to be carried. According to Article 148(3), in matters that require unanimity, deliberate absence (boycott) of a member is interpreted as a 'no' vote, which amounts to a veto; but abstention of a member (whether present or represented by another member) counts as a 'yes'.

[11] At present there are only two types of resolution that require approval by a prescribed proportion of an *entire* house. Senate Rule XXII (as amended by Senate Resolution 4 in 1975) requires that in order to invoke cloture (and thus limit debate) at least three-fifths of *all* Senate members (that is, currently at least 60 senators) must approve. Similarly, House Rule XXVII provides that any bill before a committee longer than 30 days may be brought before the House without committee approval, if a majority of the *entire* House (that is, currently at least 218 members) sign a petition that demands such action. This rule prevents a committee or a committee chairman from 'bottling up' by failure to report a bill upon which the House desires to vote.

stipulates that overriding the veto requires the approval of 'two-thirds of [each] House'; but it fails to specify explicitly whether this means two-thirds of *all* members or just of those participating in the division. However, the latter interpretation was upheld by the US Supreme Court on January 7, 1919 (*Missouri Pacific Railway Co. v. State of Kansas*, 248 U.S. 276). Specifically, the Supreme Court ruled:

> "House", within Article 1, Section 7, Clause 2, of the Constitution, requiring a two-thirds vote of each house to pass a bill over a veto, means not the entire membership, but the quorum by [Article 1] Section 5 given legislative power.[12]

In their opinion the justices quoted Mr Reed, Speaker of the House of Representatives, who had ruled in 1898 that:

> The question is one that has been so often decided that it seems hardly necessary to dwell upon it. The provision of the Constitution says, "two-thirds of both Houses", what constitutes a house? ... [T]he practice is uniform that ... if a quorum is present the House is constituted, and two-thirds of those voting are sufficient in order to accomplish the object.[13]

How then do writers on voting power report these well-established facts, upon which it seemed in 1898 'hardly necessary to dwell'? The astonishing answer is that they mis-represent them. As a typical example, let us quote from Alan Taylor's recent book (Taylor 1995, p. 46)

> **The United States Federal System**
> There are 537 voters in this yes–no voting system: 435 members of the House of Representatives, 100 members of the Senate, the vice president, and the president. The vice president plays the role of tie-breaker in the Senate, and the president has veto power that can be overridden by a two-thirds vote of both the House and the Senate. Thus, for a bill to pass it must be supported by either:
>
> 1. 218 or more representatives and 51 or more senators (with or without the vice president) and the president.
> 2. 218 or more representatives and 50 senators and the vice president and the president.

[12]See Supreme Court Reporter (1920, p. 93).

[13]Ibid., p. 95.

3. 290 or more representatives and 67 or more senators (with or without either the vice president and the president).

This description is of course incorrect, as it disregards abstentions. Now, Taylor is by no means a particularly careless reporter—quite the contrary.[14] And he is certainly in illustrious company. Thus Shapley (1962, p. 59) states bluntly: 'For example, the 1962 House of Representatives (when voting on ordinary legislation) = M_{437}.' In Shapley's notation M_n is the SVG with n voters in which the winning coalitions are those having more than $n/2$ members. On the following page Shapley displays the formula

$$\text{“Congress”} = M_{101} \times M_{437},$$

which he interprets in plain words as 'majority in both houses needed to win'.[15]

An intelligent Extra-Terrestrial visitor, presented with Shapley's report on the decision rule in the US Congress (and with no other evidence) would have to conclude that in order for ordinary legislation to pass in each of the two Houses, it needs the 'yes' of over half the *membership* of each House. This is patently false.

The hapless ET would not be disabused if he, she or it read also other scholars' writings on voting power—for example, Shapley and Shubik (1954, p. 789); Brams (1975, p. 192); Lucas (1982, p. 212); Lambert (1988, p. 235); Brams, Affuso and Kilgour (1989, p. 62) and several others. All have misrepresented the decision rule of the US legislature by implying—using plain words (like Taylor) or words and symbols (like Shapley)—that a Representative or Senator who does not vote 'yes' counts as voting 'no'.

The mis-representation of the US legislature as an SVG by Shapley and Shubik (1954) is particularly tantalizing. For, in discussing the Vice President's tie-breaking function (p. 788) they are perfectly aware that an absence of a member of the Senate during a division counts as neither 'yes' nor 'no'; and they expressly state that 'in the passage of ordinary legislation, ... perfect attendance [in the Senate] is unlikely even for important issues ...'. Yet in the very next paragraph (p. 789), when applying their index to the US legislature, they revert to the mis-statement of the decision rule:

[14] As we shall see, he is one of the two exceptional authors we were able to find who do not misrepresent the facts about the UNSC.

[15] Since 1920, the number of members of the House of Representatives has been kept fixed at 435; so the '437' in Shapley's text is attributable to the at-large representatives given in the 86th and 87th Congresses (1959–1962) to Alaska and Hawaii (which joined the US on 3 January 1959 and on 2 August 1959, respectively). Following redistricting in 1962 the number of members in the House of Representatives has been reinstated to 435 as of the 88th Congress (1963).

> It takes majorities of Senate and House, with the President, or two-thirds majorities of Senate and House without the President, to enact a bill. We take *all* [our emphasis] the members of the three bodies and consider them voting ... .

The case of the UNSC is broadly similar, but here the tale has an interesting additional twist. During the period 1945–1965 the UNSC consisted of 11 members—five permanent members and six others. In 1966 the number of non-permanent members was increased from six to 10. The (original) Article 27 of the UN Charter stated:

> (1) Each member of the Security Council shall have one vote.
>
> (2) Decisions of the Security Council on procedural matters shall be made by an affirmative vote of seven members.
>
> (3) Decisions of the Security Council on all other matters shall be made by an affirmative vote of seven members including the concurring votes of the permanent members; ... .

In 1966, when the UNSC was enlarged, the word 'seven' in clauses (2) and (3) was replaced by 'nine'. Ostensibly, the wording of Article 27(3) of the Charter implies that in non-procedural matters an explicit 'yes' vote by all permanent members is needed to pass a resolution. However, in practice, as of 1946 an explicit declaration 'I abstain' by a permanent member is not interpreted as a veto; and as of 1947 and 1950 the same applies to non-participation in the vote and absence, respectively, of a permanent member.[16] So on non-procedural matters a resolution is carried in the UNSC if it is supported by at least nine (or, before 1966, seven) members and not explicitly opposed by any permanent member. Abstention by a non-permanent member has the same effect as a 'no' vote; but abstention by a permanent member is definitely a *tertium quid*. The rule is therefore essentially *ternary*, and cannot be faithfully represented as an SVG. However, this impossibility does not seem to deter most of the scholars writing on voting power. As a typical misstatement of the facts let us quote Lambert (1988, p. 230):

> The present United Nations Security Council has 15 members. There are five major powers who are permanent members plus 10 other countries whose membership rotates. Nine votes are needed for approval of an issue, and each of the five major powers has a

[16]For details on the interpretation in practice of Article 27(3) of the UN Charter with respect to abstention, non-participation or absence of a permanent member, see Simma (1982, pp. 447–454) and references cited therein.

> veto. Thus passage of an issue requires the assent of the major five and four others.

Lambert then proceeds to represent the UNSC as a WVG in which each big power has weight 7, each non-permanent member has weight 1, and the quota needed for passing a resolution is 39. Not a word about abstention. Again, Lambert is in illustrious company. Shapley (1962, p. 65), writing before the enlargement of the UNSC, says:

> A somewhat more surprising example, since the voting strengths are not explicit in the rules, is the United Nations Security Council. The reader will readily verify that the following weights and quota accurately [*sic!*] define the voting system, complete with vetoes:
>
> $$B_5 \times M_{6,2} = [27; 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1].$$

Similar mis-statements are made by Rapoport (1970, pp. 218–219); Coleman (1971, pp. 274, 283); Brams (1975, pp. 182–191); Lucas (1982, p. 196); Riker (1982, p. 52); Brams, Affuso and Kilgour (1989, p. 58) and others.[17] It almost seems as though the Social Choice fraternity lives in an ivory tower where they can read the UN Charter but not the daily press.[18]

Note that here we are no longer concerned with opinions regarding the rationality of abstention or the desirability of taking it seriously in the theory of voting power. Nor are we concerned with how the US legislature and the UNSC *ought* to make their decisions in a perfectly rational world. We are

[17] As far as the UNSC is concerned, Bolger (1993, p. 319) was probably the first to comment explicitly on the widespread mis-reporting of the decision rule; see Addendum below.

We also wish to note that Taylor (1995, p. 46), although he presents the UNSC decision rule as an example of an SVG, is nevertheless aware that this presentation is inaccurate, and adds in parentheses: 'For simplicity, we ignore abstentions.' A similar attitude is perhaps implicit in the cautious formulation by Straffin (1982, p. 269).

[18] Anyone following press or TV reports on UNSC proceedings is in a position to notice that resolutions (on non-procedural matters) are often adopted without the assent of at least one permanent member. In the period 1946–97, this happened in the case of 300 resolutions—well over 28% of the total 1068 resolutions adopted by the UNSC. On 15 December 1973, Resolution 344 was carried by the votes of the non-permanent members, with *all five* permanent members abstaining.

In particular, the US has long made it a firm rule never to vote for any resolution condemning Israel; but occasionally such resolutions are adopted, with the US abstaining.

And some of the authors cited must be old enough to remember that the Soviet representative was absent when the UNSC decided to involve the UN in the Korean war.

concerned with reports about the way these bodies actually do make their decisions.

How can one explain what appear to be blatant factual errors made by a whole group of eminent scholars? Astonishing as this may be, phenomena of this sort are by no means exceptional in science, according to some philosophers of science, who refer to them as 'theory-laden observation': scientists often 'see' what their theory conditions them to expect.[19] In this they are indeed like ordinary folk; theory-laden observation has been compared to the commonplace phenomenon of optical illusion: we are 'deceived' by our senses into perceiving what our experience and (usually unconscious) suppositions lead us to expect.[20]

Notice that, according to the hypothesis we are proposing here, the neglect of abstention is not attributed to stupidity or ignorance. Indeed, several of the authors mentioned above have published papers and books on various topics in the field of social choice, in which they do recognize and discuss abstention as a distinct option. It is only in the context of the theory of voting power that they ignore abstentions or apparently forget all about them. In our view, the best explanation of this is that the binary theoretical SVG model with which they approach the facts predisposes them to become easy victims, *in this particular context*, of the mental counterpart of optical illusion.

Speaking for ourselves, we are not claiming to be cleverer, or better informed, than all those authors—among whom are some of the greatest scholars in the field. We can attest that so long as we worked within the SVG paradigm these factual misrepresentations, which we encountered in the literature, did not evoke in us more than a vague feeling of malaise. It is only *after* we had invented, partly by chance, the alternative ternary theoretical model, which does admit abstentions (see Felsenthal and Machover, 1997), that we became acutely aware of that widespread distortion. Now, being equipped with this model, we suddenly realized that many of the factual reports on decision rules that one encounters in the literature on voting power are seriously flawed.

Addendum

Quite a long time after submitting our paper for inclusion in this volume, we came across Bolger's paper (1993), which we had previously overlooked. In

[19]Cf. Hanson (1958) and Kuhn (1962). Perhaps a more fitting term is 'theory-*biased* observation'.

[20]Cf. Gillies (1993).

his paper Bolger defines (N, r) games, a generalization of cooperative games, in which each player is allowed to choose one of r alternatives. For $r = 2$ the alternatives can be 'yes' and 'no', so that an $(N, 2)$ game can be an SVG. For $r = 3$, a third alternative can be abstention. Bolger then proceeds to define a generalization of the Shapley value for (N, r) games. On the very first page of his paper, as a first example of an (N, r) game, he presents the decision rule of the UNSC, which he states correctly. He then adds, in a parenthetical remark,

> It should be noted that the U.N. Security council game is often erroneously modeled as a 2-alternative, namely 'yes' or 'no', game in which an issue passes if and only if it receives 'yes' votes from all five permanent members and at least 4 nonpermanent members.

It seems to us that this lends some support to the hypothesis proposed in this section. Bolger, who has a theoretical framework that allows for abstention as a distinct option, is not only able to observe the decision rule of the UNSC without distortion, but also notices that many others had got it wrong.

On the other hand, some doubt now seems to be cast on this hypothesis by our findings in Felsenthal and Machover (2000), in which we examine accounts of the US Congress and UNSC decision rules given in introductory textbooks on American Government and International Relations. It transpires that mistaken or misleading accounts are also quite widespread in this literature, to which the hypothesis of theory-laden observation cannot apply.

4 Ternary voting games

In Felsenthal and Machover (1997) we define a type of structure called a *ternary voting game* (briefly, *TVG*), which is the direct ternary analogue of an SVG: in addition to the two options of voting 'yes' or 'no', each voter may exercise a third option, abstention. We define appropriate generalizations or analogues of the Bz and S-S voting-power indices for TVGs and investigate some of their properties. In particular, we determine for each n the most 'responsive' TVGs (that is, those with a maximal sum of absolute Bz values) with n voters. Here we shall confine ourselves to some general remarks.

Finding the correct ternary analogue of an SVG is not difficult. Also, the definition of an absolute (and hence also relative) Bz index for such structures is quite straightforward; in this respect Fishburn's work (1973, pp. 53–55) pointed the way.

Using a more appropriate model can have a very significant effect on the numerical results. For example, using the unsuitable SVG model for the

UNSC, Straffin (1982, pp. 314–315) finds that $\beta = 0.1669$ for each of the five permanent members and $\beta = 0.0165$ for each of the 10 non-permanent members. But if one calculates the relative Bz indices while viewing the UNSC, more appropriately, as a TVG, one obtains $\beta = 0.1009$ for each of the five permanent members and $\beta = 0.0495$ for each of the 10 non-permanent members. Thus the more realistic TVG model ascribes to each non-permanent member of the UNSC a much greater relative a priori voting power than does the SVG model.

It could be argued that since abstention by a non-permanent member counts in practice as a 'no' vote, these members have in effect two voting options—'yes' and 'no'; whereas only the permanent members have three distinct options. The results obtained for the UNSC according to this 'mixed' SVG/TVG model are $\beta = 0.1038$ for each of the five permanent members, and $\beta = 0.0481$ for each of the 10 non-permanent members. These results are much closer to those of the pure TVG model than to those of the pure SVG model.

Using the same mixed SVG/TVG model, we also get quite different results from those obtained by Coleman (1971) regarding the *power of the UNSC to act*. According to Coleman's definition, the *power to act* is the a priori probability that a bill will be passed. Using the (inappropriate) SVG model, Coleman finds that the power of the UNSC to act was 0.0278 in the pre-1966 period, and that it *decreased* to 0.0259 post-1966 (cf. Coleman 1971, Table 1, p. 284). In the mixed SVG/TVG model, we obtain 0.03164 for the pre-1966 period, and an *increase* to 0.03619 thereafter.

Finding the right generalization of the S-S index is less easy. The most common representation of this index for SVGs imagines all voters lining up, in a random order, to vote 'yes' until a 'pivotal' voter tips the balance and the bill in question is adopted. The value ϕ_a of the S-S index for voter a is then the probability that a is that pivotal voter. This does not provide a clue as to how the S-S index may be generalized to the ternary case. However, there is another representation—stated (without proof) by Mann and Shapley (1964, p. 153)[21] —that lends itself easily and naturally to generalization. For another approach, see Bolger (1993).

While the generalization of the S-S index to TVGs is of obvious technical interest, it may be argued that it is of limited applicability. This is because, as pointed out in Section 2, the underlying justification of the S-S index is as a measure of a voter's expected relative share in a fixed purse, the prize of power. But in cases where voting can be regarded in this way (as office-

[21]For a proof, see Felsenthal and Machover (1996).

seeking behaviour) the argument that abstention is not rational does carry some weight. In our view this issue and, more generally, the status of the S-S index for both SVGs and TVGs requires some further study.[22]

The study of voting power in situations where abstention is a distinct option is in its infancy. We believe that its further development is both interesting and useful.

[22]In this connection see Felsenthal, Machover and Zwicker (1998). It should be pointed out here that it is simply a misconception to suppose, as some authors seem to do, that the justification for using the S-S index depends on its representation in terms of permutations, so that use of this index is legitimate in cases where the order in which voters cast their votes is important. The representation in terms of permutations is just that—a representation. The S-S index depends for whatever justification it may have on its being a special case of the Shapley value, whose justification, in turn, derives not from this or that representation but from its general mathematical properties and in particular from its characterization by Shapley's axioms (see Shapley, 1953).

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