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David Hume's no-miracles argument begets a valid No-Miracles Argument

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Abstract

Hume's essay 'Of Miracles' has been a focus of controversy ever since its publication. The challenge to Christian orthodoxy was only too evident, but the balance-of-probabilities criterion advanced by Hume for determining when testimony justifies belief in miracles has also been a subject of contention among philosophers. The temptation for those familiar with Bayesian methodology to show that Hume's criterion determines a corresponding balance-of-*posterior* probabilities in favour of miracles is understandable, but I will argue that their attempts fail. However, I show that his criterion generates a valid form of the so-called No-Miracles Argument appealed to by modern realist philosophers, whose own presentation of it, despite their possession of the probabilistic machinery Hume himself lacked, is invalid.

Keywords: Hume, miracles, testimony, No-Miracles Argument, probability, Bayes's Theorem.

Introduction

In his essay 'Of Miracles' forming section X of the *Enquiry* (1748), Hume enunciated this maxim (he calls it a 'general maxim worthy of our attention'):

> that no testimony is sufficient to establish a miracle, unless the testimony be of such a kind, that its falsehood would be more miraculous, than the fact, which it endeavors to establish (1748, p.115-116)

This seems to assert a necessary condition: that *only if* the testimony's falsity would be more miraculous than the occurrence of the miracle testified to, can the miracle's occurrence be taken to be established by the testimony. But shortly after this passage Hume makes it clear that he regards the condition as both necessary and sufficient:

> if the falsehood of [an individual's] testimony would be more miraculous, than the event which he relates; then, and not till then, can he pretend to commend my belief or opinion. (X, Part II)

Hume famously – for most contemporary Christians, infamously - exploited his maxim to justify the rejection of all testimony-based claims of miracles, possibly the main pillar of support for faith, since according to him the possibilities for the testimony to be false, because the testifiers were lying, deceived or otherwise mistaken, vastly outweigh the minuscule likelihood he claimed for a miracle[1]: 'we may establish it as a maxim, that no human testimony can have such force as to prove a miracle, and make it a just foundation for any such system of religion' (X, Part II).

That triumphant dismissal of testimony-based miracles has since been subjected to a great deal of critical comment. Above all, there is the question of the status of the maxim itself. Is it valid, and if so why? The issue has been hotly debated by philosophers pretty much since Hume's essay was published, and still remains the subject of philosophical controversy[2]. But the matter has turned out to be not so simple after all, with scholarly opinion equally divided over the form the parsing should take, and over whether the maxim is actually a valid thesis of probability theory. In Part I of this paper I will argue that the attempts to prove that it is a theorem of probability theory all fail, but that the relation between the two types of probability he points to, the prior probability of a hypothesis and the probability of the evidence on the assumption that the hypothesis is false, plays a crucial role in evaluating the probative power of evidence. In Part II I will show that in neglecting one of those two factors whose importance Hume had highlighted, a highly influential modern argument for scientific realism, as the No-Miracles Argument, is fallacious. I will also show that the inequality that figures in his maxim is the key to a valid and important no-miracles argument.

Part I

1. Balancing probabilities

Directly following Hume's statement of his maxim, he informs us of the inferential mechanism by which he arrived at it:

> When anyone tells me, that he saw a dead man restored to life, I immediately consider with myself, whether it be more probable, that this person should either deceive or be

[1] Hume inferred the extreme smallness of P(M) from his definition of a miracle as an event which violates the laws of nature (X, Part 1): as such, according to him, it merits a minuscule probability given the vast and varied experience on which those laws are based. Hume's critics were not slow to point out that even granted his distinctive definition, it does not follow that the prior probability of a miracle must be regarded as minute: the Catholic Church, for example, views it as quite the normal thing for God to intervene in this way given suitably justifying circumstances. And Hume's claim that experience warrants denying a miracle anything but a negligible probability is strongly in tension, to put it mildly, with his celebrated sceptical arguments in the *Enquiry* that to claim that anything is learned by experience involves the claimer in a vicious circularity..

[2] Though according to Boswell, even Dr Johnson was convinced of its correctness, if not of the conclusion Hume drew from it:

> Talking of Dr. Johnson's unwillingness to believe extraordinary things I ventured to say, 'Sir, you come near Hume's argument against miracles, "That it is more probable witnesses should lie, or be mistaken, than that they should happen." JOHNSON. 'Why, Sir, Hume, taking the proposition simply, is right. But the Christian revelation is not proved by the miracles alone, but as connected with prophecies, and with the doctrines in confirmation of which the miracles were wrought.' (1791, p.194)

> deceived, or that the fact, which he relates, should really have happened. I weigh the one miracle against the other; and according to the superiority, which I discover, I pronounce my decision, and always reject the greater miracle (X, Part I)

That inferential mechanism is thus a simple decision-rule based on a corresponding balance of probabilities. After considering which probability 'weighs' the greater - of the testimony being false (because the testifier is deceived or deceiving) against the probability, considered independently of the testimony, of the miracle having occurred - one should, according to Hume, reject the alternative with the lesser probability and accept that with the greater. In using this balance of probabilities to reject or accept the miracle's occurrence, Hume seems to have thought it equivalent to balancing the probabilities of the miracle occurring versus it not occurring, possibly reasoning thus: the testimony is false just in case the miracle did not occur; hence weighing the probability that the testimony is false against the probability of the miracle occurring is simply weighing the probability that the miracle did not occur against the probability that it did.

The reasoning may seem plausible but it is fallacious. 'The testimony is false' is not logically equivalent to 'the miracle did not occur': the left-hand side contains information about a testimony being made while the right-hand side does not, and indeed we have seen that for Hume the probability of the testimony being false is sensitive to the likelihood of alternative 'non-miraculous' explanations (e.g. the alleged witnesses were deceiving or being deceived). Hence the probability that the testimony is false cannot simply be equated with the probability of the miracle's non-occurrence, and the chain of inferences 'the probability that the testimony is false is less than (greater than) the independent probability of the miracle' => 'the probability that the miracle did not occur is less than (greater than) the probability that it did' => reject (accept) the hypothesis of the miracle's occurrence' is broken at the first link. If the decision to accept or reject the occurrence of a miracle *in the light of testimony* is to reflect a balance of the probabilities of occurrence and non-occurrence in the way Hume seems to have thought, then it is clear (at any rate post-Bayes) that those probabilities have to be *posterior probabilities given that testimony*. But Hume did not have access to the conceptual apparatus required to make that distinction: it was only just being developed by his contemporary, the mathematician and clergyman Thomas Bayes, around the time Hume was writing, in work not published until after Bayes's death in 1763 and of which the scholarly consensus is that Hume knew nothing.[3]

Comfortably post-Bayes we, unlike Hume, are in a position to answer the question he could not: does an inequality between the probability that the testimony is false and the prior probability of

[3] Earman claims that even if Hume had known of Bayes's work it is unlikely that he would have understood it (1998, p.25). That might be true for Bayes's derivation of the posterior distribution of a binomial parameter which makes up the major part of his paper, but there is little doubt that Hume could have followed Bayes's derivation of the probability axioms without difficulty, employing as it does only elementary arithmetic (it is essentially a piece of so-called Dutch Book reasoning which anticipates by two and a half centuries de Finetti's).

the miracle translate into a corresponding inequality between the posterior probability that the miracle occurred, given the testimony, and the posterior probability that it did not? Curiously enough, it was only late in the twentieth century that the question seems to have been addressed and an answer offered – indeed, more than one answer. That given by the Bayesian analysis of Gillies (1991) is a partial affirmative: the probability that the testimony is false is less than the prior probability of the miracle *if* the posterior odds on the miracle exceed the posterior odds against. However if, as Sobel (1987) and Howson (2000) assume, P(T|M) = 1, then the 'if' becomes 'if and only if'. Earman's analysis (1998) gives a fully affirmative answer but at the cost, as he himself admits, of turning Hume's maxim into a triviality (1998, p.41).

In part I of this paper I will argue that all these answers are incorrect. In so doing I will make extensive use of Bayes's Theorem in its possibly less familiar odds form (because in that form it leads to a simplified treatment), so that is where we shall start.

2. <u>Bayes's Theorem</u>.

Let 'P(X)', where X is any event/proposition, signify the probability of X, relative to whatever background information is being assumed. Some authors write this as P(X|K), where K refers to that information. Since K occurs uniformly, however, there is no need for its explicit mention and so it will be regarded as implicit in the symbolism.

Bayes's Theorem is the classic Bayesian tool for evaluating the probability of a hypothesis H in the light of evidence E. An elementary consequence of the probability axioms, it assumes a particularly simple form when expressed in terms of odds. When P(H) lies strictly between 1 and 0 the unconditional odds on H, Odds(H), defined as the quotient P(H)/P(~H), where ~H signifies the negation of H, with a corresponding quotient P(H|E)/P(~H|E) for Odds(H|E), the *conditional odds* on H given E. Odds (H|E) are often called the *posterior odds on H*, and Odds(H) the *prior odds on H*. In odds form, Bayes's theorem is just this:

$$\text{Odds}(H|E) = L(H|E).\text{Odds}(H),$$

where L(H|E) is the ratio P(E|H)/P(E|~H) of so-called likelihoods. If we idealise, as is commonly done in this sort of discussion, and assume that H actually entails E modulo the background information implicit in P, then P(E|H) = 1 and we infer that

$$\text{Odds}(H|E) = \text{Odds}(H)/P(E|{\sim}H)^{4}$$

whence we obtain immediately the biconditional

$$\text{Odds}(H|E) > q \text{ iff } \text{Odds}(H)/q > P(E|{\sim}H). \qquad (1)$$

In what follows I shall assume that all the prior probabilities lie strictly between 0 and 1, including that of a miracle. Sobel models Hume's claim that the prior probability of a miracle is

[4] It follows that H has only to predict E for P(H|E) to exceed P(H). According to Bayesians, this constitutes a partial justification of inductive reasoning.

negligible by assigning it an infinitesimal probability, where an infinitesimal is a number smaller in absolute value than every positive real number. It was proved in the mid-twentieth century that it is consistent to assume the existence of such numbers, and reciprocally-infinitesimal numbers, extending the real number field; the members of any such extension – there are infinitely many - are called hyperreals. However, since the hyperreals obey the same arithmetical rules as the reals, assigning a miracle an extremely small real probability makes little practical difference for Bayes's Theorem calculations.

3. Probability theory and Hume's maxim

Any Bayesian investigation of the validity of Hume's maxim will of course need a plausible translation of the probabilities mentioned in Hume's maxim into the formal language of modern probability theory. In what follows let T be the statement that testimony was given claiming that the miracle occurred, and M the proposition that the miracle occurred as claimed. The probability of the miracle given the testimony is a straightforward Bayesian posterior probability P(M|T), and the probability of the miracle independently of T is its prior probability P(M). This leaves the probability that the testimony is false to be parsed formally. Surprisingly, that apparently straightforward task has seen the widest degree of scholarly dissension in Bayesian discussions of Hume's maxim. Three candidates to date have been proposed: Gillies (1991), in common with Sobel (1987) and Howson (2000), advances P(T&~M) (in my notation), while Earman's gloss, proposed in the course of his sustained attack on Hume's argument (1998), is P(~M|T) (again, in my notation). The third, advanced by Dawid and Gillies (1989), is P(T|~M).[5] Which is correct?

I believe the third is correct, and my argument for it will piggyback on my answer why Earman's is not. Earman's is not because as a representation of the probability of the testimony being false P(~M|T) addresses the probability of the wrong event, M: by contrast, the question 'how likely is T *given* ~M?', i.e. 'what is P(T|~M)?', does seem to convey the correct sense, an opinion implicitly endorsed by Hume himself who cited, as relevant to assessing that probability, the possible *causes* for the testimony to be false:

> When anyone tells me, that he saw a dead man restored to life, I immediately consider with myself, whether it be more probable, that this person should either deceive or be deceived, or that the fact, which he relates, should really have happened.

In other words, the probability of the falsity of the testimony is a function of the probabilities of alternative explanations of the testimony being given other than the miracle being genuinely

[5] Since T is itself a part of background knowledge K it might be thought that P(T) must be 1, trivially rendering P(T|~M) = 1, P(M|T) = P(M) and P(~M|T) = P(~M). The question of how to deal with this apparent problem has generated an extensive sub-literature under the heading 'The Old Evidence Problem'). This is not the place to discuss it, so I will only state my opinion, and what seems to be that of the other authors offering probabilistic analyses of Hume's argument, which is that T should be counterfactually excluded from K for the purpose of the discussion (for a supporting argument, see Howson and Urbach 2006, final chapter.).

witnessed. Now a simple exercise in the probability calculus shows that $P(T|\sim M)$ does indeed convey this idea, for it is proportional to the average of the probabilities of the testimony being given relative to the alternative possible explanations of it – the deception of the testifier or they themselves being deceived, as Hume puts it – weighted by their own probabilities. In the statistical analyses of clinical treatments, where Y is a 'yes' test-result for the presence of a disease D, $P(Y|\sim D)$ always represents the probability of T being a *false positive*, which is of course just what a testimony to the truth of M is when M is not actually true.

But if $P(T|\sim M)$ is the correct parsing then Gillies's and Sobel's $P(T\&\sim M)$ must, like Earman's, also be wrong. That it is wrong is independently supported by noting that, as the unconditional probability of the testimony being given *and* the miracle not occurring, it will, other things being equal, increase or decrease with the prior probability of the testimony being given[6]. Thus $P(T\&\sim M)$ is equal both to the product $P(\sim M|T)P(T)$ and, assuming that $P(T|M) = 1$[7], to the difference $P(T) - P(M)$[8]. In each of the two cases, keeping the other factor constant, the multiplicative in the first case and the additive in the second, simply increasing the prior probability of the testimony being given would increase the probability of its falsity on this parsing. Hence that parsing cannot be right.

In what follows I will assume that, for the reasons given, $P(T|\sim M)$ is the correct formal rendering of the probability that the testimony given is false. Unfortunately for Hume's maxim, the inequality $P(T|\sim M) < P(M)$ is *not* equivalent to $P(\sim M|T) < P(M|T)$: it is easily seen to be sufficient for $P(\sim M|T) < P(M|T)$, but it is not necessary. I conclude, therefore, that the aim of exhibiting Hume's inequality $P(T|\sim M) < P(M)$ as necessary and sufficient for establishing the occurrence of M, in the sense of raising its odds above 1, fails. It might anyway be objected that allowing a hypothesis as important as that of a miracle occurring to be established with posterior odds arbitrarily close to 1 from above is an unacceptably lenient interpretation of 'established'. Even the Roman Catholic Church's criteria, not an obvious patent of authority in epistemological matters, are more demanding (or so it believes). The 'more likely than not' criterion is employed in civil cases, but the 'beyond reasonable doubt' criterion for criminal convictions would arguably be the appropriate one in a matter where human lives, to say nothing of immortal souls, could hang in the balance. One might argue that at the very least the condition for M to be 'established' on the basis of T should be that the posterior odds are *bounded away from 1* by some suitably substantial margin, i.e. there is a positive number k of a suitable magnitude such that the posterior odds exceed 1+k. 'Suitable magnitude' is of course very vague, but it is still good enough to show that the condition $P(T|\sim M) < P(M)$ is not sufficient for M to be established

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[6] Earman also points this out (1998, p.40), dismissing P(T) as irrelevant to the probability of the testimony being false.

[7] Given that Hume's discussion concerns the testimony of supposed *witnesses*, the assumption that $P(T|M)$ can be set equal to 1, or as near to 1 as makes no difference seems fully warranted: given that the miracle did occur, these people would be practically certain to report it faithfully.

[8] Since $P(T) = P(T\&\sim M)+P(T\&M) = P(T\&\sim M)+P(T|M)P(M) = P(T\&\sim M)+P(M)$.

according to this more stringent condition (Odds(M|T) only exceeds 1+k if k is sufficiently *small*). In fact, as I will show in the following section, the condition P(T|~M) < P(M) implies only that Odds(T|M) exceeds 1+Odds(M) where, of course, for Hume Odds(M), and hence P(M)[9], are minuscule.

4. A Humean induction theorem

Hume's maxim may not itself be a theorem, but his intuition that the inequality P(T|~M) < P(M) is a crucial factor in determining the degree of confidence it is proper to invest in the truth of M in the light of T is nevertheless correct. Indeed, again assuming that P(E|H) = 1, the following little theorem is easily provable from (1) and the fact that Odds(H)/[1 + Odds(H)] = P(H):

$$P(E|\sim H) < P(H) \Leftrightarrow \text{Odds}(H|E) > 1 + \text{Odds}(H)$$

In other words, the inequality P(E|~H) < P(H) is the condition not only for the posterior odds to exceed 1, but for the difference between the posterior and prior odds to exceed a fixed number (1) independent of both. We already know that satisfaction of the inequality P(E|~H) < P(H) is sufficient for the posterior odds on H to exceed 1 given P(E|H) = 1, but this result tells us much more. In particular, it tells us that so long as the prior probability of H is not negligible, then if the inequality is satisfied the posterior probability of H can be quite considerable. So, for example, if P(E|~H) is less than P(H) = ½, P(H|E) will be in excess of 2/3. So significant in the light of these observations is the inequality P(E|~H) < P(H) that I shall call it *Hume's Inequality*.

In the next section we will fast-forward more than two centuries to find that Hume's insight about the significance of the relation between the likelihood P(E|~H) and the prior P(H) does not seem to have been shared by the advocates of a twentieth-century no-miracles argument, who unlike Hume know the basic rules of probability but nevertheless believe that *all* that is needed to catapult Odds(H|E) (well) past 1 is that P(E|~H) be very small.

Part II

1 The No-Miracles Argument

Hume's declaration that he 'always rejects the greater miracle' when balancing the probabilities of competing hypotheses was echoed two and a half centuries later by John Worrall, in the course of advancing what has, unsurprisingly, come to be called the No-Miracle Argument (henceforward NMA):

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[9] Small enough odds are approximately equal to small probabilities. The odds for a given probability p are given by the function $f(p) = p/(1-p)$. Expanding about $p = 0$ we have $f(p) = p + O(p^2)$.

> It would be a miracle, a coincidence on a near-cosmic scale, if a theory made as many correct empirical predictions as, say, the general theory of relativity or the photon theory of light without what the theory says about the fundamental structure of the universe being correct or "essentially" or "basically" correct. *But we shouldn't accept miracles*, not at any rate if there is a non- miraculous alternative . . . . So it is plausible to conclude that presently accepted theories are indeed "essentially" correct. (1996, p.140; my emphasis)

But the inference drawn in the last sentence is fallacious, since it might be a greater miracle, in Humean language, for H to be true, in which case, according to Worrall, there would then be more reason to reject the 'essential' correctness of the theory itself. Without knowing how miraculous or otherwise the latter is, no inference as to its 'essential' correctness can be drawn at all. The rest, as Hamlet said, is – or should be – silence. Nor is Worrall the only offender in this matter; Popper is another:

> *it cannot be just due to an improbable accident* if a hypothesis is again and again successful when tested in different circumstances, and especially if it is successful in making previously unexpected predictions . . . If a theory *h* has been well-corroborated, then it is highly probable *that it is truth-like*. (1983, p.346; emphasis in the original)

Perhaps 'the No-Miracles Fallacy' would be a better name for this argument.

But to acknowledge what was clear to Hume, namely the need to balance the miraculousness of the agreement with the data against that of the theory, means acknowledging the indispensable role played by prior probabilities in evaluating empirical success. However, the No-Miracles argument is the inference–rule of choice for non-, even anti-, Bayesians of whom Popper is of course one *par excellence*, who claim for it an objective character free of the taint of subjectivism they see epitomised in prior probability distributions.[10] Yet what other than subjectivism informs the evaluation of P(E|~H) as extremely small and P(H) as non-negligible in the frequently-cited example of the prediction by QED to eleven places of decimals of the magnetic moment of the electron? It is logically a trivial matter to manufacture an infinity of mutually incompatible alternatives that are also in agreement with E (think *grue(t)*), and there seems no good reason to believe that it is extremely improbable that in the fullness of time some alternative explanation of the phenomenon will not be accepted. In a celebrated passage Hilary Putnam declared that 'the positive argument for realism is that it is the only philosophy that doesn't make the success of science a miracle.' (1975, p.73). Putnam is surely wrong in suggesting that the approximate truth of the theories in mature science (whose terms typically refer[11]) is the *only* explanation which does not make the latter's success a miracle. Be that as it may, the fact remains that without appeal to prior odds, or probabilities, the NMA remains invalid.

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[10] See the recent attempt by another defender of the NMA, Psillos (2009) and the reply by Howson (2013).
[11] *Ibid*.

2. A valid no-miracles argument

The little theorem proved in Part I section 4 shows that the inequality I called Hume's Inequality in a tribute to his prescience, is a sufficient condition for the posterior odds on H given E to exceed the prior odds on H by an amount greater than 1 if E is predicted by H.[12] This subsumes a valid Humean no-miracles argument, since it tells us that if P(H) is very small, then if the agreement of H with the evidence would be even more improbable ('even more miraculous') were H false than H's own truth, then we can infer that H is 'established' to the extent of being more probable than not. As far as no-miracles arguments are concerned that is, to partially quote another poet, if not all ye know on earth, at any rate pretty much all ye need to know.

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[12] Where H is the hypothesis that the theory in question is only "essentially" correct, the prediction of E by H, and the consequent support of H by E are correspondingly weakened: we now have $P(E|{\sim}H) < P(H) \Leftrightarrow Odds(H|E) > P(E|H)[1 + Odds(H)]$.

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