Anti-evasion auditing policy in the presence of common income shocks*

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Abstract

When fairly homogeneous taxpayers are affected by common income shocks, a tax agency's optimal auditing strategy consists of auditing a low-income declarer with a probability that (weakly) increases with the other taxpayers' declarations. Such policy generates a coordination game among taxpayers, who then face both strategic uncertainty - about the equilibrium that will be selected.and fundamental uncertainty - about the type of agency they face. Thus the situation can be realistically modelled as a global game that yields a unique and usually interior equilibrium which is consistent with empirical evidence.

Results are also applicable to other areas like regulation or welfare benefit allocation.

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1 Introduction

It is common practice for tax agencies worldwide to use observable characteristics of taxpayers to partition the population into fairly homogeneous categories in order to better estimate their incomes: all other things being equal, those who declare well below the estimate are likely to be evaders and are audited, while those who declare about or above it are likely to be compliant taxpayers and are not inspected. But this "cut-off" auditing policy (Reinganum and Wilde (1985)) can lead to systematic mistargeting in the presence of common shocks: in good years the category would be under-audited (bars and pubs in a heat-wave); in bad years it would be over-audited (chicken-breeders in an avian-flu outbreak).

The present article focuses on the problem a tax agency faces when deciding its auditing policy within each audit category in such scenario. To avoid systematic mistargeting, the government needs *contemporaneous* data correlated with the common shock. I examine the possibility of using the profile of declarations of the category as a signal of the shock experienced by them and show that, for a government facing a low-income declarer, the optimal auditing strategy is (weakly) increasing in the other taxpayers' declarations. Intuitively, these declarations, the more likely the shock was a positive one, and so the more likely that someone who declares low income is an evader. Precisely this type of reasoning is presumed (Alm and McKee (2004)) to be behind the method used by the IRS's "Discriminant Index Function" (DIF) to determine which taxpayers to audit.[1]

This policy introduces a negative externality among taxpayers: if someone increases her declaration, everyone else's probability of detection is increased. This changes the nature of the evasion problem by creating a coordination game among agents: each one of them has incentives to evade if most other people evade as well, and prefers to comply if most of the rest are compliant. The resulting multiplicity of equilibria (and its associated policy design problems) is avoided by the presence of an information asymmetry in favour of the tax agency. A government's innate "toughness" with respect to evasion is a parameter that is its private information, enters its objective function and affects its optimal policy: *ceteris paribus*, tougher agencies will audit more intensively than softer ones. Since this parameter is an agency's private information, taxpayers need to estimate it in order to decide how much income to declare and they do it based on the information available to them, namely, their incomes and their signals. Each taxpayer's previous experiences, conversations with friends and colleagues and interpretation of news media constitute noisy signals of the government's type and are private information. The heterogeneity of signals makes different taxpayers perceive their situations as different from other taxpayers', and yet every one of them follows the same income declaration strategy. This leads to the survival of only one equilibrium in

[1] In page 301, they say: *"(...) a taxpayer's probability of audit is based not only upon his or her reporting choices, but also upon these choices relative to other taxpayers in the cohort. In short, there is a taxpayer-taxpayer game that determines each individual's chances of audit selection."*

which (usually) some people evade and others comply, a result that is empirically supported and yet unlikely to be predicted by other tax evasion models.

Previous research on the area (started by Allingham and Sandmo (1972) and surveyed by Cowell (1990) and Andreoni et al. (1998)) did analyse the effect of asymmetric information in the tax compliance game. Some only considered the presence of "strategic uncertainty" (i.e., the uncertainty that taxpayers face in coordination games about which equilibrium will be selected), usually generated by psychological and/or social externalities (Benjamini and Maital (1985), Fortin et al. (2004), etc.). Others restricted their attention to the "fundamental uncertainty" faced by the taxpayers with respect to the type of agency (Scotchmer and Slemrod (1989), Stella (1991), etc.). The present study, on the other hand, considers both types of uncertainty and thus models the situation as a global game (Carlsson and van Damme (1993), Morris and Shin (2002b)).

The closest references to the present article are Alm and McKee (2004), Basseto and Phelan (2004) and Kim (2005). The first one is a laboratory experiment where the (*ad hoc*) auditing policy is contingent on the distribution of income declarations, while the second and third use the global game technique to determine the optimal tax system and the auditing policy, respectively. This paper presents a theoretical analysis in which –unlike the laboratory experiment– the agency's optimal strategy is derived instead of assumed. The other two studies employ the same technique that I use here, but while Basseto and Phelan (2004) is concerned with the optimal tax system as designed by a government, this article focuses only on the targeting aspect of *one* of the agencies of the government. Finally, Kim (2005) generates the strategic interaction among taxpayers by adding a "stigma cost" to their utility functions, whereas in my case it is the result of a cunning tax agency that sets its auditing policy to maximise its objective function.

2 Model

The model focuses on the interaction between the tax agency (also referred to as "the government") and the taxpayers (or "agents") *within a given category*. For simplicity, I will use "population of taxpayers" and "common shocks" to indicate the members of the category and the shocks faced by them, and *not* those of the whole population (i.e., the set which is the union of all the categories), unless indicated otherwise.

The timing of the game is as follows: First, actors receive their pieces of private information (the agency its type λ, the taxpayers their incomes y and signals s). Then taxpayers submit their income declarations d and pay taxes accordingly. Finally, the agency undertakes audits and collects fines (if any).

2.1 Taxpayer problem

Taxpayer i's problem consists of choosing how much income to declare in order to maximise her expected utility.

Taxpayers are uniformly distributed on the $[0,1]$ segment and their income is their own private information. Agents are assumed to be risk-neutral, so that their utility is a linear function of their disposable income:

$$u_i = y_i - td_i - a_i \cdot f_i \qquad \forall i \in [0,1] \tag{1}$$

where $y_i \in \{0,1\}$ is agent i's gross (taxable) income, $t \in (0,1)$ is the income tax rate, $d_i \in \{0,1\}$ is agent i's income declaration, $a_i \in \{0,1\}$ is an indicator function defined as

$$a_i = \begin{cases} 1 & \text{if agent } i \text{ is audited} \\ 0 & \text{if agent } i \text{ is not audited} \end{cases} \tag{2}$$

and f_i is the fine agent i should pay if audited, defined as

$$f_i = \begin{cases} (1+\varsigma)\, t\, (y_i - d_i) & \text{if } d_i < y_i \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

($\varsigma \in (0,1)$ is the surcharge rate that has to be paid by a caught evader on every dollar of evaded taxes).

Taxpayers know all the parameters of the problem. They also know the probability distributions of the other players' private information (the agency's type λ and other taxpayers' signals $s_{j \neq i}$), though they do not know their realizations. Taxpayers' incomes are assumed perfectly correlated to reflect the fact that common shocks affect similar agents in similar ways: in "good years" (which occur with probability $\gamma \in (0,1)$) everyone gets a high income ($y_i = 1\ \forall i \in [0,1]$) and in "bad years" (which occur with probability $1-\gamma$) everyone gets a low income ($y_i = 0\ \forall i \in [0,1]$). Accordingly, all this knowledge constitutes the taxpayer's information set, I_i.

In order to decide how much income to declare, a taxpayer i needs to estimate as accurately as possible the auditing policy of the agency with respect to herself, a_i. Since the decision on a_i is made by the agency *after* all tax returns are submitted (i.e., after it observes the vector of income declarations $\mathbf{d}$), taxpayers know that the audit decision will be a function of the declaration profile $\mathbf{d}$ and the agency's type λ, and so they will estimate them using all their available information: $E\left[a_i\left(\mathbf{d}, \lambda\right) \mid I_i\right]$.

One of the elements included in taxpayers' information sets, private signals convey information about the government's type and are, on average, correct. They reflect the information about the agency's type that taxpayers get from all available sources: media news, previous experiences, conversations with colleagues and friends, etc. Formally,

$$s_i := \lambda + \varepsilon_i \qquad \forall s_i \in S \tag{4}$$

where S is the signal's domain and ε_i is the error term, which is assumed to be white noise ($E(\varepsilon_i) = 0 \ \forall i$), uniformly distributed on the $[-\varepsilon, \varepsilon]$ segment, and independent of income y_i, other taxpayers' errors $\varepsilon_{j \neq i}$ and the government's type λ.

The taxpayer problem is therefore

$$\max_{\{d_i\}} \qquad E\left[u\left(d_i, a_i\left(d_i, \mathbf{d}_{-i}, \lambda\right)\right) \mid y_i, s_i\right] \tag{5}$$

which can be re-written as

$$\max_{\{d_i\}} \qquad y_i - t d_i - f_i \cdot E\left[a_i\left(d_i, \mathbf{d}_{-i}, \lambda\right) \mid y_i, s_i\right] \tag{6}$$

The optimal declaration will be a function of the taxpayer's income and her (subjective) probability of detection if she evades, $d^*\left(y_i, E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid y_i, s_i\right]\right)$. Hence, two cases need to be considered: one when $y_i = 0$ and the other when $y_i = 1$. In both, agent i has to decide whether to declare low ($d_i = 0$) or high income ($d_i = 1$).

The first case is straightforward and is characterised in the following proposition:

Proposition 1 *In bad years ($y_i = 0 \ \forall i \in [0,1]$) taxpayers always declare truthfully. Formally,*

$$d^*\left(0, E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 0, s_i\right]\right) = 0 \qquad \forall s_i \in S \mid \lambda \tag{7}$$

where $S \mid \lambda$ is the signal's domain conditional on the value of the agency's type.

Proof. From the comparison of the expected utilities an agent with low income ($y_i = 0$) gets when she declares low income ($E\left[u\left(0, a_i\left(0, \mathbf{d}_{-i}, \lambda\right)\right) \mid 0, s_i\right] = 0$) and when she declares high income ($E\left[u\left(0, a_i\left(1, \mathbf{d}_{-i}, \lambda\right)\right) \mid 0, s_i\right] = -t$). ■

In good years (i.e., when $y_i = 1$), a low-income declaration leads to an expected utility of $E\left[u\left(0, a_i\left(0, \mathbf{d}_{-i}, \lambda\right)\right) \mid 1, s_i\right] = 1 - (1 + \varsigma)\, t \cdot E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 1, s_i\right]$, while a high-income one yields $E\left[u\left(1, a_i\left(1, \mathbf{d}_{-i}, \lambda\right)\right) \mid 1, s_i\right] = 1 - t$.

Taxpayers' decisions depend on the comparison between the two as follows

$$d^*\left(1, E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 1, s_i\right]\right) = \begin{cases} 0 & \text{if } E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 1, s_i\right] < P \\ \in [0,1] & \text{if } E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 1, s_i\right] = P \\ 1 & \text{if } E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 1, s_i\right] > P \end{cases} \tag{8}$$

where $P := \frac{1}{(1+\varsigma)}$ is the probability of detection that eliminates evasion.

Intuitively, in good years taxpayers evade only if their subjective belief about the probability of being audited is not too high. This implies that an agent's declaration is (weakly) increasing in her expectation over the probability of detection.

Combining the results for bad and good years (proposition 1 and equation 8), the solution to the taxpayer problem is

$$d^*\left(y_i, E\left[a_i\left(0\right) \mid y_i, s_i\right]\right) = \begin{cases} 0 & \text{if } y_i = 0 \\ 0 & \text{if } y_i = 1 \text{ and } E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 1, s_i\right] < P \\ \in [0,1] & \text{if } y_i = 1 \text{ and } E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 1, s_i\right] = P \\ 1 & \text{if } y_i = 1 \text{ and } E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid 1, s_i\right] > P \end{cases} \tag{9}$$

from which it is clear that an agent's declaration is (weakly) increasing in her gross income.

The latter results are summarised in the following proposition:

Proposition 2 *A taxpayer's optimal declaration strategy is:* (1) *(weakly) increasing in her (subjective) expectation over the probability of detection* $E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid y_i, s_i\right]$*, and* (2) *(weakly) increasing in her gross income* y_i*. Formally,*

$$(1) \quad \frac{\partial d^*\left(y_i, E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid y_i, s_i\right]\right)}{\partial E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid y_i, s_i\right]} \geqslant 0 \qquad (2) \quad \frac{\partial d^*\left(y_i, E\left[a_i\left(0, \mathbf{d}_{-i}, \lambda\right) \mid y_i, s_i\right]\right)}{\partial y_i} \geqslant 0 \tag{10}$$

Proof. By direct inspection of equation 9. ■

Further characterizing a taxpayer's optimal declaration strategy, the next proposition shows how it is influenced by private signals:

Proposition 3 *In good years, a taxpayer's optimal declaration strategy:* (1) *is a step function,* (2) *is (weakly) increasing in her private signal* s_i*, and* (3) *it is the same for all taxpayers. Formally,*

$$(1) \quad d^*\left(1, s_i\right) = \begin{cases} 0 & \textit{if } s_i < \hat{s} \\ \in [0,1] & \textit{if } s_i = \hat{s} \\ 1 & \textit{if } s_i > \hat{s} \end{cases} \qquad (2) \quad \frac{\partial d^*\left(1, s_i\right)}{\partial s_i} \geqslant 0 \tag{11}$$

where $\hat{s} := \tilde{\lambda} + \frac{4+\alpha}{5}\varepsilon\left(2P-1\right)$*,* $\alpha \in (0,1)$ *and* $\tilde{\lambda} := 1 - \gamma$*.*

Proof. For the first part, see appendix. For the second part, by direct inspection of equation 11.1. For the third part, it is the result of $\hat{s}$ being a constant that is independent of the identity of the taxpayer whose strategy is being studied. ■

The intuition is straightforward: the higher the signal received ($s_i := \lambda + \varepsilon_i$ from equation 4), the higher is the taxpayer's (subjective) expectation over the government's type

λ, meaning that the agent believes that, very likely, she faces a tough agency and, thus, a high probability of detection. This decreases the (subjective) expected return of evasion and makes compliance more attractive, which leads the taxpayer to (weakly) increase her income declaration.

The final part of the proposition highlights the fact that, though having different private signals, all taxpayers agree on the "switching point" below one should evade and above which one should comply. This result is going to be used later on in order to find the equilibrium of the game.

Note also that, as expected, each "type" of taxpayer (defining agent i's "type" as its private information pair (y_i, s_i)) has a unique optimal strategy: taxpayers with low income ($y_i = 0$) ignore their signals and always declare low income; taxpayers with high income ($y_i = 1$) do take into account the signals they receive and declare income as shown in equation 11.1.

2.2 Tax agency problem

Narrowly defined, a tax agency's objective is to raise revenue. More generally, its problem consists of determining which citizens should be audited and which ones should not.

An agency, therefore, chooses its auditing strategy in order to minimise its targeting errors.[2]

These errors can be of two types: Negligence and Zeal. A negligence mistake occurs when a "profitable audit" is not undertaken. A zeal error takes place when an "unprofitable audit" is carried out.

An audit is defined as "profitable" if the fine obtained if undertaken more than compensates for the cost of carrying it out (formally, if $f_i > c$, with f_i being the fine –as defined in equation 3– and $c \in (\varsigma t, (1+\varsigma)\, t)$ the cost of the audit). It is assumed that an audit that discovers an evader is always profitable, while an audit that targets a compliant taxpayer is always unprofitable. Formally, if $\alpha_i = 1$ means that auditing agent i is profitable, then

$$\alpha_i := \begin{cases} 1 & \text{if } y_i = 1 \text{ and } d_i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Hence, a negligence error (N_i) occurs when the audit is profitable ($\alpha_i = 1$) and it is not undertaken ($a_i = 0$). On the other hand, a zeal error (Z_i) occurs when the audit is not

[2]The analysis also holds if the "expected net revenue" (taxes plus fines minus enforcement costs) is used as the agency's objective function.

profitable ($\alpha_i = 0$) and yet it is undertaken ($a_i = 1$). Formally,

$$N_i := \begin{cases} 1 & \text{if } \alpha_i = 1 \text{ and } a_i = 0 \\ 0 & \text{otherwise} \end{cases} \qquad\qquad Z_i := \begin{cases} 1 & \text{if } \alpha_i = 0 \text{ and } a_i = 1 \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

For the rest of the article, and due to the fact that they make the problem more tractable, I will use –without loss of generality– the following two error functions:

$$N_i := (1 - a_i)(1 - d_i)\, y_i \qquad\qquad Z_i := a_i \left[1 - (1 - d_i)\, y_i\right] \tag{14}$$

Different agencies can, however, value each kind of error differently. If $\lambda \in \Lambda$ is defined as the weight attached to negligence errors, the loss inflicted by agent i on an agency of type λ can be expressed as

$$L_i := \lambda N_i + (1 - \lambda)\, Z_i \tag{15}$$

The parameter λ is the agency's "type" and it is assumed to be its private information. Henceforward, I will call "tough" those agencies with high values of λ (which bear a high loss when an evader is not caught) and "soft" those with low values of λ (which bear a high loss when a compliant taxpayer is audited).

The government knows all the parameters of the problem and its own private information (its type λ). It does not know taxpayers' incomes or signals, though it does know their probability distributions. More importantly, it observes the vector of declarations $\mathbf{d}$, and can therefore make its auditing policy contingent on it. Accordingly, all this knowledge constitutes its information set, I_G. Conditional on it, the agency's estimated loss from agent i is

$$E_G\left[L_i\right] := E\left[L_i\left(y_i, d_i\left(y_i, s_i\right), a_i\left(d_i, \mathbf{d}_{-i}\right)\right) \mid I_G\right] = E\left[\lambda N_i + (1 - \lambda)\, Z_i \mid \mathbf{d}, \lambda\right] \tag{16}$$

which can be re-expressed as

$$E_G\left[L_i\right] = \lambda\left(1 - a_i\left(\mathbf{d}\right)\right) \cdot E_G\left[(1 - d_i)\, y_i\right] + (1 - \lambda) \cdot a_i\left(\mathbf{d}\right) \cdot \left\{1 - E_G\left[(1 - d_i)\, y_i\right]\right\} \tag{17}$$

The aggregate expected loss is therefore

$$E\left[L\left(\mathbf{y}, \mathbf{d}\left(\mathbf{y}, \mathbf{s}\right), \mathbf{a}\left(\mathbf{d}\right)\right) \mid \mathbf{d}, \lambda\right] := \int_{s_i \in S \mid \lambda} E_G\left[L_i\right]\; dG\left(s_i \mid \lambda\right) \tag{18}$$

where $S \mid \lambda$ is a signal's domain, conditional on the value of λ, and $G\left(s_i \mid \lambda\right)$ is the cumulative probability distribution of agent i's signal conditional on the agency's type being λ (this distribution is consistent with equation 4 and the paragraph immediately after it).

The agency's problem is to choose the auditing strategy $\mathbf{a}(\mathbf{d})$ as to minimise the aggregate expected loss. Formally,

$$\min_{\{\mathbf{a}(\mathbf{d})\}} E\left[L\left(\mathbf{y}, \mathbf{d}\left(\mathbf{y}, \mathbf{s}\right), \mathbf{a}\left(\mathbf{d}\right)\right) \mid \mathbf{d}, \lambda\right] \tag{19}$$

The solution to this problem depends on the actual profile of declarations $\mathbf{d}$ observed by the agency, which means that there are 3 interesting cases to consider: 1. when everyone declares high income ($\mathbf{d} = \mathbf{1}$, $\mathbf{1} := (1, ..., 1)$), 2. when some people declare high income and others declare low income ($\mathbf{d} \neq \mathbf{1}$ and $\mathbf{d} \neq \mathbf{0}$, $\mathbf{0} := (0, ..., 0)$), and 3. when everyone declares low income ($\mathbf{d} = \mathbf{0}$).[3] The results are summarised in the following proposition:

Proposition 4 *For every taxpayer, a λ-type agency's optimal auditing strategy is as follows:*

$$a_i^*\left(d_i, \mathbf{d}_{-i}, \lambda\right) = \begin{cases} 0 & \text{if } d_i = 1 \\ 0 & \text{if } d_i = 0,\ \mathbf{d} = \mathbf{0},\ \text{and } \lambda < \tilde{\lambda} \\ \in [0,1] & \text{if } d_i = 0,\ \mathbf{d} = \mathbf{0},\ \text{and } \lambda = \tilde{\lambda} \\ 1 & \text{if } d_i = 0,\ \mathbf{d} = \mathbf{0},\ \text{and } \lambda > \tilde{\lambda} \\ 1 & \text{if } d_i = 0,\ \text{and } \mathbf{d} \neq \mathbf{0} \end{cases} \tag{20}$$

where $\tilde{\lambda} := 1 - \gamma$ and $\gamma \in (0,1)$ is the probability of a good year.

Proof. In the appendix. ■

Intuitively, the proposition says that an agency's optimal auditing decision with respect to a given taxpayer i depends on the taxpayer's decision d_i, the declarations of all other taxpayers $\mathbf{d}_{-i}$, and the agency's type λ. When at least one person declares high income (and so $\mathbf{d} \neq \mathbf{0}$), the government knows for sure –thanks to the perfect correlation assumption– that the shock was a positive one (it was a "good year"), and so the optimal strategy consists of auditing everyone who declares low income ($a_i^*\left(0, \mathbf{d}_{-i} \neq \mathbf{0}, \lambda\right) = 1$, since they are evaders) and not auditing anyone who declares high income ($a_i^*\left(1, \mathbf{d}_{-i}, \lambda\right) = 0$, since only "rich" taxpayers ever declare high income, and so their declarations are truthful). When everyone declares low income (and so $\mathbf{d} = \mathbf{0}$), the government cannot tell whether it faces a population of "poor" compliant taxpayers or one of "rich" evaders. The optimal policy therefore depends on how tough the government is (i.e., how high λ is) and how likely it is for the taxpayers to face a good year (i.e., the value of γ). If the agency is rather tough (λ is rather high), the optimal policy consists of auditing everyone (and the same is true if the probability of a good year, γ, is high). Otherwise (if the agency is rather soft or a bad year is very likely), it is better for the agency to audit no one.

[3] As a desirable feature of the agency's optimal auding strategy, I will impose the condition that it should be "*ex-post* horizontally equitable", that is, that identical agents should be treated equally. In this setting, it means that those who declare the same income should be either *all* audited or *not one of them* audited by the agency. Formally,

$$a_i\left(d_i = k\right) = a_j\left(d_j = k\right)\ \ \forall i, j, k$$

These results are summarised in the following proposition:

Proposition 5 *For every taxpayer, a λ-type agency's optimal auditing strategy is:* (1) *(weakly) increasing in the agency's type λ, and* (2) *(weakly) increasing in the probability of a good year γ. Formally,*

$$(1) \quad \frac{\partial a_i^*(d_i,\mathbf{d}_{-i},\lambda)}{\partial \lambda} \geqslant 0 \qquad\qquad (2) \quad \frac{\partial a_i^*(d_i,\mathbf{d}_{-i},\lambda)}{\partial \gamma} \geqslant 0 \tag{21}$$

Proof. By direct inspection of equation 20. ■

Further characterizing the agency's optimal strategy, the next result describes how it depends on the taxpayer's own declaration as well as on every other taxpayer's declarations:

Proposition 6 *For every taxpayer, a λ-type agency's optimal auditing strategy is:* (1) *(weakly) increasing in every other taxpayers' declaration $d_{j\neq i}$, and* (2) *(weakly) decreasing in the taxpayer's own declaration d_i. Formally,*

$$(1) \quad \frac{\partial a_i^*(d_i,\mathbf{d}_{-i},\lambda)}{\partial d_{j\neq i}} \geqslant 0 \qquad\qquad (2) \quad \frac{\partial a_i^*(d_i,\mathbf{d}_{-i},\lambda)}{\partial d_i} \leqslant 0 \tag{22}$$

Proof. By direct inspection of equation 20. ■

Intuitively, this means that the agency audits individuals who declare high income with a lower probability than those who declare low income (as is standard in tax evasion models). The novelty of the present study is in the result of equation 22.1, which shows that a loss-minimising agency would use the information conveyed by the vector of income declarations (or the average declaration, which in this case is a sufficient statistics) when deciding its optimal policy. In particular, the declarations of other taxpayers provide *contemporaneous* information about the likelihood of a given income shock, improving the targeting proficiency of the agency that can thus perfectly distinguish between truthful and untruthful declarations when the profile of declarations is different from **0**.

The latter result has a crucial effect on the whole tax evasion game and, in combination with that of equation 10.1, makes taxpayer i's optimal declaration strategy a (weakly) increasing function of the other taxpayers' declarations:

Proposition 7 *Taxpayers' declarations are (weakly) strategic complements. Formally, for every $j \neq i$,*

$$\frac{\partial d_i^*\left(y_i, s_i\right)}{\partial d_{j\neq i}} \geqslant 0 \tag{23}$$

Proof. Directly from propositions 2 and 6. ■

This proposition opens a second channel through which a higher signal leads to a higher declaration (in addition to the one described in proposition 3): a high signal means that other taxpayers are also likely to receive high signals –and to declare high income too– which increases the expected probability of detection and makes compliance relatively more attractive (i.e., provides incentives to (weakly) increase the amount of income declared).

Even more importantly, this result transforms the nature of the tax evasion problem, because it creates a *coordination game* among the taxpayers *on top of* the cat-and-mouse game that each one of them plays against the agency and that is usually the only one considered by the literature. The strategic complementarity between taxpayers' declaration, however, is not an inherent characteristic of the game, but rather one that is *created* by the agency in its attempt to minimise its targeting errors. Indeed, it is the fact that the auditing strategy is an increasing function of other taxpayers' declarations (Proposition 6) that creates the strategic complementarity. That is, a cunning agency, willing to minimise its targeting-related losses, designs its optimal auditing strategy by introducing some *strategic uncertainty* (i.e., by creating a coordination game between taxpayers) that improves its ability to distinguish compliant from non-compliant agents and thus decreases the occurrence of targeting mistakes.

3 Equilibrium

A priori, the generation of a coordination game among taxpayers does not look as a good idea for the agency because this kind of games present multiple equilibria, which make policy design a complicated matter. Nevertheless, this difficulty is overcome thanks to the presence of a second source of uncertainty (called "fundamental uncertainty") that allows for the tax evasion problem to be modelled as a "global game" (Carlsson and van Damme (1993), Morris and Shin (2002b)).[4]

This equilibrium-selection technique eliminates all but one equilibria owing to the introduction of some heterogeneity in taxpayers' information sets in the form of the noisy private signals they receive and that convey information about the government's private information parameter λ (the source of the "fundamental uncertainty"). Thus, taxpayers do not observe the true coordination game (as they would do if signals were 100% accurate), but slightly different versions of it. This is the case since taxpayers with different signals would work out different estimates of the agency's type λ and other people's declarations $\mathbf{d}_{-i}$, and so of their probabilities of detection. The optimal declaration strategy, however, is one and

[4] In other applications (bank runs, currency crises, etc (Atkeson (2000))), this technique has been criticised because of not taking into account the coordinating power of markets and prices. This criticism is greatly mitigated in the case of tax evasion, since there is no "insurance market against an audit" to aggregate information about the government's type (the "fundamental", in global games jargon).

the same for every "type" of taxpayer (propositions 1 and 3). The rationale for this result goes along the lines described in the paragraph immediately after the proof of proposition 7: my own signal gives me information about the possible signals that other taxpayers may have received and, more importantly, about the signals that they *cannot* have received, thus allowing me to discard some strategies that they cannot have followed. The application of this process iteratively by *every* taxpayer leads to the elimination of all strictly dominated strategies and leaves only one optimal strategy to be followed by every taxpayer (Morris and Shin (2002a)), namely, the ones in propositions 1 and 3.

As a consequence, once the private information variables (the agency's type λ and taxpayers' incomes and signals $(\mathbf{y}, \mathbf{s})$) are realised, the equilibrium will be *unique*.

However, depending on the value of λ, the equilibrium can present different features, as shown in the following proposition:

Proposition 8 *The unique equilibrium of the tax evasion game looks like one of the following cases:* (1) Full evasion *($\lambda < \hat{s} - \varepsilon$): in good years, every taxpayer evades and nobody is audited,* (2) Partial evasion *($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$): in good years, taxpayers with low signals ($s_i < \hat{s}$) evade and are audited with certainty while those with high signals ($s_i > \hat{s}$) comply and are not audited, and* (3) Full compliance *($\hat{s} + \varepsilon < \lambda$): in good years, every taxpayer complies and everyone who declares low income is audited. In bad years, every taxpayer declares truthfully in all three cases. Formally,*

	Full evasion	*Partial evasion*	*Full compliance*
$d^*(0, s_i)$	0	0	0
$d^*(1, s_i)$	0	$\begin{cases} 0 & \text{if } s_i < \hat{s} \\ \in [0,1] & \text{if } s_i = \hat{s} \\ 1 & \text{if } s_i > \hat{s} \end{cases}$	1
$a_i^*(d_i, \mathbf{d}_{-i}, \lambda)$	0	$\begin{cases} 0 & \text{if } d_i = 1 \\ 1 & \text{if } d_i = 0 \end{cases}$	1

(24)

Proof. Follows directly from the optimal strategies of the players (propositions 1, 3 and 4) and the characterisation of the equilibrium in terms of the average declaration (proposition 9 below). ■

Since in bad years taxpayers declare low income in every scenario, the three cases are characterised (and labelled) according to the actions taken by taxpayers in good years. The full evasion case occurs when the agency is so soft ($\lambda < \hat{s} - \varepsilon$) that all taxpayers know it will audit nobody who declares low income, and so everyone evades. The opposite occurs in the full compliance case, in which the agency is so tough ($\hat{s} + \varepsilon < \lambda$) that all taxpayers know it will audit everyone who declares low income, and so everyone complies. The partial evasion case occurs when the government is not too soft nor too tough ($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$)

and so rich taxpayers cannot tell for sure whether everyone else will evade or will comply, though all of them would like to do as most taxpayers do (*strategic complementarity*). They therefore follow the optimal strategy described in proposition 3, which means that the average declaration will be greater than zero. The agency, observing this, would know for sure that true income is high and so will audit everyone who declares 0 and nobody that declares 1.

A straightforward corollary of the previous proposition is the one that links the average declaration (and so the level of evasion) and the government's type:

Proposition 9 *In bad years ($y_i = 0\ \forall i \in [0,1]$), the average declaration is zero ($\bar{d}^* = 0$), as is the level of evasion ($\kappa^* = 0$). In good years ($y_i = 1\ \forall i \in [0,1]$), the corresponding values are as follows:*

	Full evasion	*Partial evasion*	*Full compliance*
Average declaration $\bar{d}^*$	0	$\frac{\lambda+\varepsilon-\hat{s}}{2\varepsilon}$	1
Level of evasion κ^*	1	$1-\frac{\lambda+\varepsilon-\hat{s}}{2\varepsilon}$	0

(25)

Proof. In the appendix. ■

This shows that, as expected, evasion is lower the tougher the government is.

Building on these results, one can further characterise the three cases:

Proposition 10 *The payoffs of the players in the three possible scenarios are as follows:*

	Full evasion	*Partial evasion*	*Full compliance*
Taxpayer/ Bad year	0	0	0
Taxpayer/ Good year	1	$\begin{cases} 1-t & \text{if } d_i = 1 \\ 1-(1+\varsigma)\,t & \text{if } d_i = 0 \end{cases}$	$1-t$
Tax Agency	$\gamma\lambda$	0	$(1-\gamma)(1-\lambda)$

(26)

Proof. Follows directly from the definition of the payoff functions of the players (equations 6 and 18), their optimal strategies (propositions 1, 3 and 4) and the characterisation of the equilibrium in terms of the average declaration (proposition 9). ■

In bad years a taxpayer's payoff is a direct consequence of her declaring truthfully her low income and getting no punishment or reward for doing so, regardless of the value of λ. The other two actors' payoffs, on the other hand, are different depending on the case under consideration. In good years, with full evasion, every taxpayer evades and, since no one is audited, each one of them keeps their gross income. In turn, since the agency audits no one, it suffers an expected loss of $\gamma\lambda$ because with probability γ the year is a good one and so everyone is an evader who is not caught (negligence errors) and with probability $1-\gamma$ the year is a bad one, everyone complies and nobody is audited (no zeal errors). Analogously, with full compliance, all taxpayers comply and so their disposable income is simply their gross income minus their voluntarily paid taxes, $1-t$. The expected loss of the agency is now $(1-\gamma)(1-\lambda)$ because with probability γ the year is a good one, everyone complies and nobody is audited (no negligence errors) and with probability $1-\gamma$ the year is a bad one and everyone complies but is audited anyway (zeal errors). The most interesting scenario is, however, the partial evasion one. Here, the agency makes no targeting error whatsoever, thus reaching the best outcome it could aspire to. The rationale behind this result is that some taxpayers will evade (those with low signals) while others will comply (those with high signals) and so the agency can perfectly distinguish evaders from compliant taxpayers, which implies that evaders are always caught (their payoffs are equal to gross income minus fine, $1-(1+\varsigma)\,t$) while compliant taxpayers are never targeted (they get payoffs equal to gross income minus taxes $1-t$). This means that the government is better off when it can create a *coordination game* among agents but, especially, when it in turn *makes taxpayers take different actions* (some evade, others comply), thus getting valuable information about the true income of the population and increasing its targeting accuracy.

To conclude the characterisation of the equilibrium, it is important to analyse how more accurate signals affect the level of evasion and the agency's payoff:

Proposition 11 *More precise information (formally, a lower ε) leads to:* (1) *(weakly) less compliance if the agency is soft ((weakly) more if it is tough), and* (2) *a (weakly) higher expected loss if the agency is soft ((weakly) lower if it is tough). Formally,*

$$(1) \quad \frac{\partial \bar{d}^*}{\partial \varepsilon} \begin{cases} \geqslant 0 & \text{if } \lambda < \tilde{\lambda} \\ \leqslant 0 & \text{if } \lambda > \tilde{\lambda} \end{cases} \qquad\qquad (2) \quad \frac{\partial EL^*}{\partial \varepsilon} \begin{cases} \leqslant 0 & \text{if } \lambda < \tilde{\lambda} \\ \geqslant 0 & \text{if } \lambda > \tilde{\lambda} \end{cases} \tag{27}$$

where $\tilde{\lambda} := 1-\gamma$.

Proof. In the appendix. ■

The proposition highlights the fact that the impact of better information depends on the type of the agency. This is at odds with previous studies, which usually find that better information is bad for the government, through the argument that less accurate information increases the risk borne by taxpayers who, assumed to be risk averse, have therefore more incentives to comply.

Though compelling, this argument cannot be applied to the present case because here agents are assumed risk neutral. Yet, what matters is that the relationship between compliance (or expected loss) and accuracy of information is not *intrinsically* (weakly) increasing or decreasing, but rather one whose shape depends on the type of the government. Intuitively, when an agency is soft (λ is low) it dislikes targeting compliant taxpayers and so would audit with a very low probability. For signals of a given precision $\varepsilon > 0$, agents will estimate the probability of detection and decide their income declarations accordingly. If the signals became more precise (if ε decreased), agents would be more aware of the fact that the agency is soft (in the extreme case, when $\varepsilon = 0$, they would know it with certainty), and so would expect a lower probability of detection, which in turn makes evasion relatively more attractive and leads to lower compliance and, accordingly, higher losses for the government. An analogous story can be used when the agency is tough (λ is high): it abhors letting evaders get away with their cheating and therefore audits with a very high probability. In this case, an increase in precision makes taxpayers more aware of the fact that the agency is tough, and so they expect a lower return for evasion due to the higher probability of detection. This, in turn, leads to an increase in compliance and a corresponding decrease in the agency's expected loss.

4 Discussion

As every other model, the one developed here is built around some simplifying assumptions that make it more tractable and elegant, but also more restrictive and unrealistic.

Indeed, it could be argued that tax agencies do not follow a "bang-bang" policy such that either everyone is audited or nobody is, but rather one where a fraction of the population is audited while the rest is not. The first approach is a direct consequence of the "*ex-post* horizontal equity" condition, while the second one would fit a situation that satisfies the condition of "horizontal equity in expectation". The former is a stronger version of the latter, but also leads to situations where those who declare equal amounts are *effectively* treated equally, a desirable feature of an optimal auditing policy in my view. However, if the second approach were used, the results would not be significantly different from the ones presented in the text, the only "major" difference being that a tough agency would not audit everyone, but rather just a fraction of the population sufficiently large as to eliminate all incentives to evade (with the added benefit that the enforcement costs will be lower due to the smaller number of audits undertaken).

Also unlikely to be found in the real world is the dichotomous character of income assumed here. When more than two levels of income are allowed, the auditing decision with respect to a given individual depends on the *relative position* of the taxpayer's declaration compared to the rest of the population's: if it is among the highest ones, then the taxpayer is audited with a given probability, usually between 0 and 1, contingent on the agency's type and decreasing in the amount declared; if it is not, the agency knows the taxpayer is lying and audits her with certainty. When only two levels of income are considered, this policy collapses to the one presented in the present article.[5]

Along similar lines, it is clear that the assumption of perfect correlation among the taxpayers' incomes is an implausible one. However, it is just intended to capture the fact that usually taxpayers that belong to the same category are homogeneous in most aspects, including income. Relaxing it will not change the (qualitative) results, as long as the common shocks are maintained as the *main source of income variability.* This ensures that there is still a significant degree of correlation among incomes and, therefore, that other taxpayers' declarations convey *useful information* about the common shock that affects the category. Even more important, what really matters for the analysis is the fact that incomes within a class are more homogeneous than the signals received by its members, such that the differences among them are mainly due to disparate perceptions of the government's type. Thus, the assumption of perfect uniformity allows observing the effect of the fundamental uncertainty unadulterated by the presence of income heterogeneity, and so the analysis is greatly simplified.

Finally, the importance of the partitioning of the taxpayer population into fairly homogeneous categories is highlighted by the fact that the above mentioned "relatively high correlation" condition is achieved when the category consists of agents that are very similar to each other in terms of their "observables" (age, profession, gender, etc.), since in this case their idiosyncratic shocks will be relatively small compared to the category-wide ones.[6] However, since the partitioning problem is an issue this paper is not concerned with, the only related matter worth discussing here is the type of classes that favours the present model. And since the latter clearly relies on some degree of uniformity within the class, its predictions are more likely to fit the data from classes with a large number of rather homogeneous people (e.g., unskilled manufacture workers or non-executive public servants) than the ones from small and/or heterogeneous classes.

5 Conclusion

The question of a tax agency's optimal auditing strategy in the presence of common income shocks is relevant because it is not unusual for such shocks to be the main source of income variability for a group of fairly homogeneous taxpayers. Under these circumstances an agency's best policy consists of auditing those who declare low income with a probability that

[5] Also, irrespective of the levels of income allowed, if they are bounded above (i.e., $y_i \leqslant y_{\max}\ \forall i \in [0,1]$), the agency would never audit those who declare $y_{\max}$. In the more realistic case of unbounded domain, the probability of detection simply decreases as the declaration increases, as is standard in the literature.

[6] These "observables" refer to variables that are exogenous to (or costly to manipulate by) the agents, and so do *not* include taxpayers' current declarations.

is (weakly) increasing in the declarations of the other taxpayers in the category. Intuitively, the higher these declarations, the more likely the shock was a positive one, and hence the more likely that someone who declares low income is an evader.

Implementing this policy does not require new information to be gathered by the agency, just using the available information better. Yet, it changes the nature of the problem for the taxpayers: on top of the standard cat-and-mouse game each one of them plays against the agency, they also play a coordination game against each other, a game they would *not* play if the policy were not contingent on the average declaration.

The heterogeneity in private signals eliminates the policy design difficulties that the multiplicity of equilibria appears to generate and paves the way for modelling the problem as a global game which not only is more realistic, but also predicts a unique equilibrium which is consistent with empirical evidence.

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A Appendix

Proof. Proposition 4

The agency problem consists of choosing an auditing strategy in order to minimise its expected loss (see equations 17 and 18). As indicated in the paragraph immediately after equation 19, three cases need to be considered:

1. $\mathbf{d} = \mathbf{1}$: Since nobody with a low income would ever declare 1 (proposition 1), the agency knows it is a good year ($\mathbf{y} = \mathbf{1}$) with certainty. The expected loss from each agent (equation 17) reduces therefore to $E_G[L_i(\mathbf{1})] = (1-\lambda) \cdot a_i(1, \mathbf{1})$, which is increasing in the audit decision $a_i(1, \mathbf{1})$, and so the agency sets it equal to zero for everyone.

2. $\mathbf{d} \neq \mathbf{1}$ and $\mathbf{d} \neq \mathbf{0}$: The agency can again infer that the year was a good one ($\mathbf{y} = \mathbf{1}$), since at least one taxpayer declared high income. The expected loss from each agent (equation 17) now becomes (after some algebraic manipulation) $E_G[L_i(\mathbf{d})] = \lambda(1 - a_i(d_i, \mathbf{d})) - (\lambda - a_i(d_i, \mathbf{d})) \cdot d_i$. Without loss of generality, assume that a fraction $\kappa \in (0,1)$ of the population declared 0 (i.e., evaded) and the remaining $1-\kappa$ declared 1 (i.e., complied). Each one of the evaders generates an expected loss of $E_G[L_i(0, \mathbf{d})] = \lambda(1 - a_i(0, \mathbf{d}))$, and each one of the compliant taxpayers generates an expected loss of $E_G[L_i(1, \mathbf{d})] = (1-\lambda) \cdot a_i(1, \mathbf{d})$. The aggregate expected loss is therefore $E_G[L(\mathbf{d})] = \kappa \cdot E_G[L_i(0, \mathbf{d})] + (1-\kappa) \cdot E_G[L_i(1, \mathbf{d})]$. This expression is a decreasing function of $a_i(0, \mathbf{d})$ and an increasing function of $a_i(1, \mathbf{d})$, and so the agency sets them equal to 1 and 0 respectively.

3. $\mathbf{d} = \mathbf{0}$: In this case the agency cannot determine with certainty whether it is a good or bad year. The expected loss from each agent (equation 17) thus becomes $E_G[L_i] = \lambda(1 - a_i(0, \mathbf{0})) \cdot E_G[y_i] + (1-\lambda) \cdot a_i(0, \mathbf{0}) \cdot \{1 - E_G[y_i]\}$. Using Bayes' rule, the government's posterior belief about the type of year conditional on knowing that everyone declared 0 reduces to the prior expectation γ, and so the expected loss per agent is now $E_G[L_i] = \lambda(1 - a_i(0, \mathbf{0})) \cdot \gamma + (1-\lambda) \cdot a_i(0, \mathbf{0}) \cdot \{1-\gamma\}$, which is

increasing in $a_i(0,\mathbf{0})$ if $\lambda < 1-\gamma$ and decreasing if $\lambda > 1-\gamma$, so that the agency would set $a_i(0,\mathbf{0})$ equal to 0 if $\lambda < 1-\gamma$, to 1 if $\lambda > 1-\gamma$ and to any value in the $[0,1]$ interval if $\lambda = 1-\gamma$.

Combining the results of the three cases, equation 20 is obtained. ■

Proof. Proposition 3

In good years, taxpayers know that if *any* one of them declares high income, every one who declares low will be audited for sure. Thus, if a taxpayer expects at least one other agent to comply, she would rather comply. This means that agent i will only evade if two conditions are met: (1) her belief about the probability of detection is sufficiently low, and (2) she expects everyone else to evade as well. Formally, (1) $E[a_i(0,\mathbf{d},\lambda) \mid s_i] < P$, and (2) $E[E[a_j(0,\mathbf{d},\lambda) \mid s_j] \mid s_i] < P \ \forall j \neq i$.

The first equation is simply the condition for evasion as presented in equation 8. The second one means that agent i expects everyone else to evade as well, that is, that she expects every other taxpayer's condition for evasion to be met as well. The two equations are therefore self-consistent if and only if they hold when $\mathbf{d} = \mathbf{0}$, that is, when everyone evades. Thus, the equations become

$$\begin{array}{lc} (1') & E[a_i(0,\mathbf{0},\lambda) \mid s_i] < P \\ (2') & E[E[a_j(0,\mathbf{0},\lambda) \mid s_j] \mid s_i] < P \ \forall j \neq i \end{array} \tag{28}$$

Consider first condition $28.1'$. The expected probability of detection conditional on agent i's information set is given by

$$E[a_i(0,\mathbf{0},\lambda) \mid s_i] = \int_{\lambda\in\Lambda|s_i} \int_{\mathbf{s}\in(S|\lambda)\times(S|\lambda)} a_i(0,\mathbf{0},\lambda)\, dG(\mathbf{s} \mid \lambda)\, dF(\lambda \mid s_i) \tag{29}$$

where $F(\lambda \mid s_i)$ is the probability distribution of the agency's type (conditional on agent i's signal taking the value s_i), $\Lambda \mid s_i$ is the domain of the agency's type (conditional on agent i's signal taking the value s_i), $G(\mathbf{s} \mid \lambda)$ is the joint probability distribution of the signals (conditional on the agency's type taking the value λ), and $(S \mid \lambda) \times (S \mid \lambda)$ is the domain of the vector of signals $\mathbf{s}$ (conditional on the agency's type taking the value λ).

Since the vector of declarations is fixed at $\mathbf{0}$, the expression simplifies to

$$E[a_i(0,\mathbf{0},\lambda) \mid s_i] = \int_{\lambda\in\Lambda|s_i} a_i(0,\mathbf{0},\lambda)\, dF(\lambda \mid s_i) \tag{30}$$

Bearing in mind that the agency's optimal strategy when everyone declares low income is as indicated in equation 20 (lines 2, 3 and 4), three cases need to be considered:

1'.1. if $\tilde{\lambda} < s_i - \varepsilon$, and since signals are uniformly distributed around λ (equation 4 and the paragraph immediately after it), taxpayer i's expected probability of detection (equation 30) is equal to $E\left[a_i\left(\mathbf{0},\lambda\right) \mid s_i\right] = \int_{s_i-\varepsilon}^{s_i+\varepsilon} (2\varepsilon)^{-1}\, d\lambda = 1$, and so greater than P. Hence, following her optimal strategy (equation 8), she will comply:

$$d_i = 1 \quad \text{if} \quad \tilde{\lambda} < s_i - \varepsilon \tag{31}$$

1'.2. if $s_i - \varepsilon < \tilde{\lambda} < s_i + \varepsilon$, the expression becomes $E\left[a_i\left(\mathbf{0},\lambda\right) \mid s_i\right] = \int_{\tilde{\lambda}}^{s_i+\varepsilon} (2\varepsilon)^{-1}\, d\lambda = \left(s_i + \varepsilon - \tilde{\lambda}\right)(2\varepsilon)^{-1}$, and so agent i will evade (assuming that equation $28.2'$ is also satisfied) only if this expression is not greater than P, that is, if and only if $s_i < \tilde{\lambda} + \varepsilon\left(2P-1\right)$. Agent i's optimal strategy in this case is therefore

$$d_i = \begin{cases} 0 & \text{if} \quad \tilde{\lambda} - \varepsilon < s_i < \tilde{\lambda} + \varepsilon\left(2P-1\right) \\ \in [0,1] & \text{if} \quad s_i = \tilde{\lambda} + \varepsilon\left(2P-1\right) \\ 1 & \text{if} \quad \tilde{\lambda} + \varepsilon\left(2P-1\right) < s_i < \tilde{\lambda} + \varepsilon \end{cases} \tag{32}$$

1'.3. if $s_i + \varepsilon < \tilde{\lambda}$, the expected probability of detection is 0, and so agent i would evade (assuming equation $28.2'$ is also satisfied):

$$d_i = 0 \quad \text{if} \quad s_i + \varepsilon < \tilde{\lambda} \tag{33}$$

Combining the three cases (equations 31, 32 and 33), the optimal strategy for agent i is (again assuming equation $28.2'$ is also satisfied):

$$d_i = \begin{cases} 0 & \text{if} \quad s_i < \tilde{\lambda} + \varepsilon\left(2P-1\right) \\ \in [0,1] & \text{if} \quad s_i = \tilde{\lambda} + \varepsilon\left(2P-1\right) \\ 1 & \text{if} \quad s_i > \tilde{\lambda} + \varepsilon\left(2P-1\right) \end{cases} \tag{34}$$

Consider now equation $28.2'$ and compute $E\left[E\left[a_j\left(0,\mathbf{d},\lambda\right) \mid s_j\right] \mid s_i\right]$. It is given by

$$E\left[E\left[a_j\left(0,\mathbf{d},\lambda\right) \mid s_j\right] \mid s_i\right] = E\left[\int_{\lambda\in\Lambda|s_j} \int_{\mathbf{s}\in(S|\lambda)\times(S|\lambda)} a_j\left(0,\mathbf{0},\lambda\right) dG\left(\mathbf{s} \mid \lambda\right) dF\left(\lambda \mid s_j\right) \mid s_i\right] \tag{35}$$

which, as before, can be simplified to

$$E\left[E\left[a_j\left(0,\mathbf{d},\lambda\right) \mid s_j\right] \mid s_i\right] = E\left[\int_{\lambda\in\Lambda|s_j} a_j\left(0,\mathbf{0},\lambda\right) dF\left(\lambda \mid s_j\right) \mid s_i\right] \tag{36}$$

Again, we need to consider three cases:

2'.1. if $\tilde{\lambda} < s_j - \varepsilon$, using equation 31, equation 36 becomes $E\left[E\left[a_j \mid s_j\right] \mid s_i\right] = E\left[1 \mid s_i\right] = 1$.

2'.2. if $s_j - \varepsilon < \tilde{\lambda} < s_j + \varepsilon$, using now equation 32, the above mentioned equation becomes $E\left[E\left[a_j \mid s_j\right] \mid s_i\right] = E\left[\frac{s_j+\varepsilon-\tilde{\lambda}}{2\varepsilon} \mid s_i\right] = \frac{E[s_j|s_i]+\varepsilon-\tilde{\lambda}}{2\varepsilon} = \frac{s_i+\varepsilon-\tilde{\lambda}}{2\varepsilon}$

2'.3. if $s_j + \varepsilon < \tilde{\lambda}$, using equation 33, we get $E\left[E\left[a_j \mid s_j\right] \mid s_i\right] = E\left[0 \mid s_i\right] = 0$.

But, unlike the case of condition $28.1'$, we cannot consider each case individually because agent i does not know the value of agent j's signal s_j (and so whether case 1, 2 or 3 is in place), while she did know her own signal s_i when dealing with condition $28.1'$. The computation of $E\left[E\left[a_j \mid s_j\right] \mid s_i\right]$ must therefore take into account the likelihood of each of the three case, i.e.,

$$\begin{aligned} E\left[E\left[a_j \mid s_j\right] \mid s_i\right] \quad = \quad & prob\left(\tilde{\lambda} < s_j - \varepsilon \mid s_i\right) \cdot 1 + \\ & prob\left(s_j - \varepsilon < \tilde{\lambda} < s_j + \varepsilon \mid s_i\right)\left(s_i + \varepsilon - \tilde{\lambda}\right)(2\varepsilon)^{-1} + \\ & prob\left(s_j + \varepsilon < \tilde{\lambda} \mid s_i\right) \cdot 0 \end{aligned} \tag{37}$$

which can be re-expressed as

$$\begin{aligned} E\left[E\left[a_j \mid s_j\right] \mid s_i\right] \quad = \quad & 1 - prob\left(s_j < \tilde{\lambda} + \varepsilon\right) + \\ & \left[prob\left(s_j < \tilde{\lambda} + \varepsilon\right) - prob\left(s_j < \tilde{\lambda} - \varepsilon\right)\right] \cdot \frac{s_i + \varepsilon - \tilde{\lambda}}{2\varepsilon} \end{aligned} \tag{38}$$

where the conditioning on the value of s_i is omitted for simplicity.

Using the definition of a signal (equation 4) for agents i and j, agent j's signal can be re-written as $s_j = s_i - \varepsilon_i + \varepsilon_j$, and so

$$\begin{aligned} E\left[E\left[a_j \mid s_j\right] \mid s_i\right] \quad = \quad & 1 - F_{\varepsilon_j - \varepsilon_i}\left(\tilde{\lambda} + \varepsilon - s_i\right) + \\ & \left[F_{\varepsilon_j - \varepsilon_i}\left(\tilde{\lambda} + \varepsilon - s_i\right) - F_{\varepsilon_j - \varepsilon_i}\left(\tilde{\lambda} - \varepsilon - s_i\right)\right] \cdot \frac{s_i + \varepsilon - \tilde{\lambda}}{2\varepsilon} \end{aligned} \tag{39}$$

where $F_{\varepsilon_j - \varepsilon_i}(x)$ is the probability distribution of $\varepsilon_j - \varepsilon_i$. Since ε_i and ε_j are random variables uniformly distributed in the $[-\varepsilon, \varepsilon]$ interval, the probability distribution of $\varepsilon_j - \varepsilon_i$ is

$$F_{\varepsilon_j - \varepsilon_i}(x) = \begin{cases} 0 & \text{if} \quad x < -2\varepsilon \\ \frac{1}{2}\left(1 + \frac{x}{2\varepsilon}\right)^2 & \text{if} \quad -2\varepsilon < x < 0 \\ 1 - \frac{1}{2}\left(1 + \frac{x}{2\varepsilon}\right)^2 & \text{if} \quad 0 < x < 2\varepsilon \\ 1 & \text{if} \quad 2\varepsilon < x \end{cases} \tag{40}$$

Thus,

$$F_{\varepsilon_j-\varepsilon_i}\left(\tilde{\lambda}+\varepsilon-s_i\right)=\begin{cases}0 & \text{if} \quad \tilde{\lambda}+3\varepsilon< \quad s_i \\ \frac{1}{2}\left(1+\frac{\tilde{\lambda}+\varepsilon-s_i}{2\varepsilon}\right)^2 & \text{if} \quad \tilde{\lambda}+\varepsilon< \quad s_i \quad <\tilde{\lambda}+3\varepsilon \\ 1-\frac{1}{2}\left(1+\frac{\tilde{\lambda}+\varepsilon-s_i}{2\varepsilon}\right)^2 & \text{if} \quad \tilde{\lambda}-\varepsilon< \quad s_i \quad <\tilde{\lambda}+\varepsilon \\ 1 & \text{if} \quad s_i \quad <\tilde{\lambda}-\varepsilon\end{cases} \tag{41}$$

and

$$F_{\varepsilon_j-\varepsilon_i}\left(\tilde{\lambda}-\varepsilon-s_i\right)=\begin{cases}0 & \text{if} \quad \tilde{\lambda}+\varepsilon< \quad s_i \\ \frac{1}{2}\left(1+\frac{\tilde{\lambda}-\varepsilon-s_i}{2\varepsilon}\right)^2 & \text{if} \quad \tilde{\lambda}-\varepsilon< \quad s_i \quad <\tilde{\lambda}+3\varepsilon \\ 1-\frac{1}{2}\left(1+\frac{\tilde{\lambda}-\varepsilon-s_i}{2\varepsilon}\right)^2 & \text{if} \quad \tilde{\lambda}-3\varepsilon< \quad s_i \quad <\tilde{\lambda}-\varepsilon \\ 1 & \text{if} \quad s_i \quad <\tilde{\lambda}-3\varepsilon\end{cases} \tag{42}$$

Now, the first two cases of equation 41 and the first of equation 42 require s_i to be large ($s_i>\tilde{\lambda}+\varepsilon$), but this would lead agent i to declare high income based on condition $28.1'$. Hence, I will concentrate on the remaining cases, which can be reduced to the following three:

2'.1'. If $s_i<\tilde{\lambda}-3\varepsilon$, then $F_{\varepsilon_j-\varepsilon_i}\left(\tilde{\lambda}+\varepsilon-s_i\right)=1$, and $F_{\varepsilon_j-\varepsilon_i}\left(\tilde{\lambda}-\varepsilon-s_i\right)=1$, and so $E\left[E\left[a_j\mid s_j\right]\mid s_i\right]=0$. Thus, agent i believes every other agent j would evade ($E\left[E\left[a_j\mid s_j\right]\mid s_i\right]=0<P$), and so she evades.

2'.2'. If $\tilde{\lambda}-3\varepsilon<s_i<\tilde{\lambda}-\varepsilon$, then $F_{\varepsilon_j-\varepsilon_i}\left(\tilde{\lambda}+\varepsilon-s_i\right)=1$ and $F_{\varepsilon_j-\varepsilon_i}\left(\tilde{\lambda}-\varepsilon-s_i\right)=1-\frac{1}{2}\left(1+\left(\tilde{\lambda}-\varepsilon-s_i\right)(2\varepsilon)^{-1}\right)^2$, and so $E\left[E\left[a_j\mid s_j\right]\mid s_i\right]=\frac{1}{2}\left(1+\left(\tilde{\lambda}-\varepsilon-s_i\right)(2\varepsilon)^{-1}\right)^2\cdot\left(s_i+\varepsilon-\tilde{\lambda}\right)(2\varepsilon)^{-1}$, which is negative, and so strictly smaller than P. Again, agent i expects everyone else to evade, and so she evades.

2'.3'. If $\tilde{\lambda}-\varepsilon<s_i<\tilde{\lambda}+\varepsilon\,(2P-1)<\tilde{\lambda}+\varepsilon$, then $F_{\varepsilon_j-\varepsilon_i}\left(\tilde{\lambda}+\varepsilon-s_i\right)=1-\frac{1}{2}\left(1+\frac{\tilde{\lambda}+\varepsilon-s_i}{2\varepsilon}\right)^2$ and $F_{\varepsilon_j-\varepsilon_i}\left(\tilde{\lambda}-\varepsilon-s_i\right)=\frac{1}{2}\left(1+\frac{\tilde{\lambda}-\varepsilon-s_i}{2\varepsilon}\right)^2$, and so

$$\begin{aligned}E\left[E\left[a_j\mid s_j\right]\mid s_i\right] \quad = \quad & 1-\left[1-\frac{1}{2}\left(1+\frac{\tilde{\lambda}+\varepsilon-s_i}{2\varepsilon}\right)^2\right]+ \\ & \left[1-\frac{\left(1+\frac{\tilde{\lambda}+\varepsilon-s_i}{2\varepsilon}\right)^2}{2}-\frac{\left(1+\frac{\tilde{\lambda}-\varepsilon-s_i}{2\varepsilon}\right)^2}{2}\right]\frac{s_i+\varepsilon-\tilde{\lambda}}{2\varepsilon}\end{aligned} \tag{43}$$

In order to make the analysis simpler, define

$$v_i:=\frac{s_i+\varepsilon-\tilde{\lambda}}{2\varepsilon}, \qquad 0<v_i<P<1 \tag{44}$$

Equation 43 can therefore be re-written as a cubic function of v_i: $E\left[E\left[a_j \mid s_j\right] \mid s_i\right] = \frac{1}{16} v_i \left[17 - (4v_i - 3)^2\right]$ (represented by the solid thick line in figure 1).

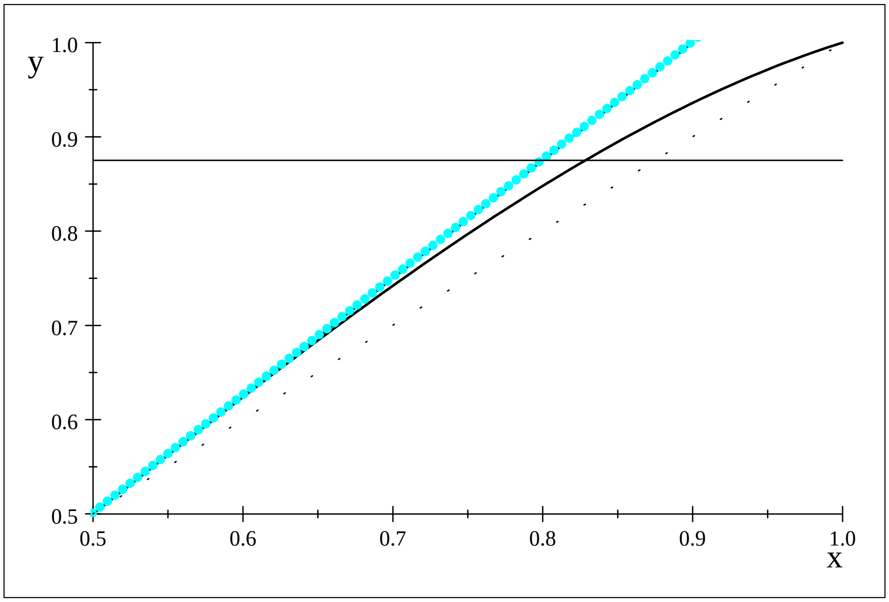

Figure 1: Horizontal axis: v_i. Vertical axis: v_i (hollow-dots line), $E\left[E\left(a_j \mid s_j\right) \mid s_i\right]$ (thick solid line), linearisation (solid-dots line), P (horizontal line at 0.875).

For agent i to evade, this expression must be below P. Define the threshold value of v_i (that is, the one that ensures $E\left[E\left[a_j \mid s_j\right] \mid s_i\right] = P$) as $\tilde{v}$. Formally,

$$\frac{1}{16} \tilde{v} \left[17 - (4\tilde{v} - 3)^2\right] := P \tag{45}$$

The threshold $\tilde{v}$ is always smaller than P: since P can only take values in the $\left(\frac{1}{2}, 1\right)$ interval, $E\left[E\left[a_j \mid s_j\right] \mid s_i\right]$ is always greater than v_i (the hollow-dots line in figure 1) in the relevant range $v_i \in \left(\frac{1}{2}, 1\right)$, and so $E\left[E\left[a_j \mid s_j\right] \mid s_i\right] = P$ for $\tilde{v} < v_i$.

In order to find a closed form solution, I linearise $E\left[E\left[a_j \mid s_j\right] \mid s_i\right]$ around $v_i = \frac{1}{2}$, and get the expression

$$E\left[E\left[a_j \mid s_j\right] \mid s_i\right] \approx \frac{1}{8} \left(10v_i - 1\right) \tag{46}$$

(the solid-dots line in figure 1). The threshold (using the linearisation in equation 46) is defined by $\frac{1}{8}\left(10v^- - 1\right) := P$, that is, $v^- := \frac{1}{10}\left(1 + 8P\right)$.

Since the linearisation (equation 46) over-estimates the true function (equation 45) for the relevant ranges of $v_i \in \left(\frac{1}{2}, 1\right)$ and $P \in \left(\frac{1}{2}, 1\right)$, then the linearised threshold underestimates the true threshold ($v^- < \tilde{v}$). This can be seen graphically in figure 1: the thresholds are found at the intersection between the different functions and the horizontal line that represents P (in the figure, assumed to take the value 0.875). It can be seen that the true function intersects this line at a higher value of v_i than the linearised function or, equivalently, that the true threshold is higher than the linearised one. Since v_i and s_i are positively related (equation 44), we can find the equivalent thresholds, $\tilde{s}$ and s^-, namely, $\frac{1}{16}\frac{\tilde{s}+\varepsilon-\tilde{\lambda}}{2\varepsilon}\left[17-\left(4\frac{\tilde{s}+\varepsilon-\tilde{\lambda}}{2\varepsilon}-3\right)^2\right] := P$ and $s^- := \tilde{\lambda}+\frac{4}{5}\varepsilon\,(2P-1)$.

Thus, the linearisation will under-estimate evasion: for agents with signals in the $(s^-, \tilde{s})$ range the linearisation advises them to comply, when they should evade. Hence, the results based on the linearisation can be used as a lower bound for the level evasion. Along the same lines, one can define P as an upper bound ($v^+ := P$ and $s^+ := \tilde{\lambda}+\varepsilon\,(2P-1)$), since, if such bound were used, agents with signals in the $(\tilde{s}, s^+)$ intervals would evade when they should comply.

The finding of upper and lower limits allows therefore to parameterise the threshold as a weighted average of the two: $\hat{v} := \alpha v^+ + (1-\alpha)\,v^- = \alpha P + (1-\alpha)\,\frac{1+8P}{10}$, or

$$\hat{s} := \alpha s^+ + (1-\alpha)\,s^- = \tilde{\lambda} + \frac{4+\alpha}{5}\varepsilon\,(2P-1) \tag{47}$$

which is the threshold level as defined in proposition 3 ($\alpha \in [0,1]$ is the weight attached to each limit: when $\alpha = 1$, evasion will be over-estimated, when $\alpha = 0$, evasion will be under-estimated). Thus, agent i would evade only if her signal is low enough, that is, if $\tilde{\lambda}-\varepsilon < s_i < \hat{s} < \tilde{\lambda}+\varepsilon$.

In summary, the analysis for conditions 28.1$'$ and 28.2$'$ (needed for agent i to declare low income) leads to the following results:

- From 28.1$'$, for agent i to evade $s_i < \tilde{\lambda}+\varepsilon\,(2P-1)$ is needed (equation 34);
- Condition 28.2$'$, in turn, requires: $s_i < \tilde{\lambda}-3\varepsilon$, $\tilde{\lambda}-3\varepsilon < s_i < \tilde{\lambda}-\varepsilon$, and $\tilde{\lambda}-\varepsilon < s_i < \hat{s}$ (cases $2'.1'.$, $2'.2'.$ and $2'.3'.$, respectively), which simplify to $s_i < \hat{s}$.

Since $\hat{s} \leqslant \tilde{\lambda}+\varepsilon\,(2P-1)$, it is straightforward to see that this leads to an optimal strategy for agent i that is identical to the one described in proposition 3. ■

Proof. Proposition 9

The average declaration is defined as

$$\bar{d} := \int_{s_i\in(S|\lambda)} d_i\,(1, s_i)\,dG\,(\mathbf{s}\mid\lambda) \tag{48}$$

which, given the taxpayer's optimal strategy in good years (equation 3), can be interpreted as the fraction of the population that gets a signal above the threshold $\hat{s}$.

Depending on the value of λ, three cases can occur:

1. Full evasion ($\lambda < \hat{s} - \varepsilon$): Even the person with highest signal (i.e., $s_i = \lambda + \varepsilon$) would evade. Formally, $\bar{d} := \int_{\lambda-\varepsilon}^{\lambda+\varepsilon} (0) \frac{1}{2\varepsilon} d\mathbf{s} = 0$.

2. Partial evasion ($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$): Those with signals between $\lambda - \varepsilon$ and $\hat{s}$ evade, those with signals between $\hat{s}$ and $\lambda + \varepsilon$ comply. Formally, $\bar{d} := \int_{\lambda-\varepsilon}^{\hat{s}} (0) \cdot \frac{1}{2\varepsilon} d\mathbf{s} + \int_{\hat{s}}^{\lambda+\varepsilon} (1) \frac{1}{2\varepsilon} d\mathbf{s} = \frac{\lambda+\varepsilon-\hat{s}}{2\varepsilon}$.

3. Full compliance ($\hat{s} + \varepsilon < \lambda$): Even the person with the lowest signal (i.e., $s_i = \lambda - \varepsilon$) would comply. Formally, $\bar{d} := \int_{\lambda-\varepsilon}^{\lambda+\varepsilon} (1) \frac{1}{2\varepsilon} d\mathbf{s} = 1$.

The level of evasion is simply the fraction of the population that –in a good year– gets a signal below the threshold $\hat{s}$. That is, $\kappa = 1 - \bar{d}$. ■

Proof. Proposition 11

Consider first the full evasion case ($\lambda < \hat{s} - \varepsilon$). Since the "generalised threshold" $\hat{s}$ is defined as in equation 47, the condition $\lambda < \hat{s} - \varepsilon$ becomes

$$\varepsilon < \frac{\tilde{\lambda} - \lambda}{2\delta} \tag{49}$$

where δ is defined as $\delta := \frac{1}{2}\left[1 - \frac{4+\alpha}{5}(2P-1)\right]$.

Since $\alpha \in [0,1]$ and $P \in \left(\frac{1}{2}, 1\right)$, δ can only take values in the interval $\left(0, \frac{1}{2}\right)$. Also, since the noise of the signals cannot be negative, it must be the case that

$$0 < \varepsilon \tag{50}$$

Combining equations 49 and 50, the full evasion case requires $0 < \varepsilon < \frac{\tilde{\lambda}-\lambda}{2\delta}$, which is only feasible if $\lambda < \tilde{\lambda}$ (i.e., full evasion is only feasible if the government is soft).

In the full compliance case ($\hat{s} + \varepsilon < \lambda$), the condition $\hat{s} + \varepsilon < \lambda$ becomes

$$\varepsilon < \frac{\lambda - \tilde{\lambda}}{2(1-\delta)} \tag{51}$$

Combining equations 51 and 50, the full compliance case requires $0 < \varepsilon < \frac{\lambda-\tilde{\lambda}}{2(1-\delta)}$, which is feasible only if $\tilde{\lambda} < \lambda$ (i.e., full compliance is only feasible if the government is tough).

Finally, the condition needed for the existence of the partial evasion case ($\hat{s} - \varepsilon < \lambda < \hat{s} + \varepsilon$) becomes $\varepsilon > \max\left\{\frac{\tilde{\lambda}-\lambda}{2\delta}, \frac{\lambda-\tilde{\lambda}}{2(1-\delta)}\right\}$. If $\lambda < \tilde{\lambda}$ it becomes $\varepsilon > \frac{\tilde{\lambda}-\lambda}{2\delta}$. If $\tilde{\lambda} < \lambda$, it is $\varepsilon > \frac{\lambda-\tilde{\lambda}}{2(1-\delta)}$.

Summarising the results so far, there are two cases to consider: (1) if the government is soft ($\lambda < \tilde{\lambda}$) the full evasion case arises when the noise is low ($\varepsilon < \frac{\tilde{\lambda}-\lambda}{2\delta}$) and the partial evasion one when it is high; and (2) if the government is tough ($\tilde{\lambda} < \lambda$) the full compliance case occurs when the noise is low ($\varepsilon < \frac{\lambda - \tilde{\lambda}}{2(1-\delta)}$) and the partial evasion when it is high.

Hence, using proposition 9, the average declaration in each of the two cases is given by

$$\begin{array}{ll} (1) \quad \bar{d}^* = \begin{cases} 0 & \text{if} \quad 0 < \varepsilon < \frac{\tilde{\lambda}-\lambda}{2\delta} \\ \delta + \frac{\tilde{\lambda}-\lambda}{2\varepsilon} & \text{if} \quad \frac{\tilde{\lambda}-\lambda}{2\delta} < \varepsilon \end{cases} \\ (2) \quad \bar{d}^* = \begin{cases} 1 & \text{if} \quad 0 < \varepsilon < \frac{\lambda-\tilde{\lambda}}{2(1-\delta)} \\ \delta + \frac{\tilde{\lambda}-\lambda}{2\varepsilon} & \text{if} \quad \frac{\lambda-\tilde{\lambda}}{2(1-\delta)} < \varepsilon \end{cases} \end{array} \tag{52}$$

It is straightforward from here to prove the first part of the proposition by simply computing the derivative of $\bar{d}$ with respect to ε.

For the second part, using the two cases considered above and proposition 10, the expected loss of the agency is as follows

$$\begin{array}{ll} (1) \quad EL^* = \begin{cases} \gamma\lambda & \text{if} \quad 0 < \varepsilon < \frac{\tilde{\lambda}-\lambda}{2\delta} \\ 0 & \text{if} \quad \frac{\tilde{\lambda}-\lambda}{2\delta} < \varepsilon \end{cases} \\ (2) \quad EL^* = \begin{cases} 0 & \text{if} \quad 0 < \varepsilon < \frac{\lambda-\tilde{\lambda}}{2(1-\delta)} \\ (1-\gamma)(1-\lambda) & \text{if} \quad \frac{\lambda-\tilde{\lambda}}{2(1-\delta)} < \varepsilon \end{cases} \end{array} \tag{53}$$

The computation of the derivative of EL^* with respect to ε yields the result. ■