Endogenous Industry Structure in Vertical Duopoly

Maija Halonen[*]
Academy of Finland and Helsinki School of Economics

The Toyota Centre
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London School of Economics and Political Science
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Abstract

We examine integration decisions of successive duopolists. We show that qualitatively the same pattern of integration emerges whether there is a Cournot or Bertrand competition in the input market. We find that the degree of integration in the industry is increasing in the size of the downstream market and decreasing in the average marginal cost of the industry and in the fixed integration cost. There is a tendency for partial integration when one upstream firms is relatively efficient compared to its rival.

Keywords: endogenous industry structure; vertical duopoly; integration; competition; market.

Contact address: Dr Maija Halonen, Department of Economics, Helsinki School of Economics, Runeberginkatu 22-24, 00100 Helsinki, Finland.

1 Introduction

We aim to develop some fairly general properties for the degree of integration in the industry. We first analyse in detail a model where there is Cournot competition in the input market. Then we compare the results to those of a model where nonlinear prices are applied in the input market (Bertrand competition). We show that in both cases qualitatively the same pattern of integration emerges.

Incentives for integration in a vertical duopoly are driven by three externalities. First, we have the vertical externality of double-marginalization. Second, a horizontal externality emerges when downstream firms compete in the product market. Third, there is an excessive supply incentive: every additional unit of input an upstream firm sells to one downstream firm reduces the profit of the other downstream firm by depressing the final good price. An unintegrated upstream firm does not take this marginal effect on the downstream firm's profit into account and therefore sells too much input to the other firm. Double-marginalization effect is proportional to the upstream firm's profit margin since that is the distortion in question. Excessive supply incentive is the greater, the higher is the downstream firm's profit margin because then the loss from increased rival's output is highest. The horizontal effect depends on both margins. High downstream margin tells that there is a large gain from expanding output. While high upstream margin means that the input cost will be much lower after integration.

In our model there are two upstream firms and two downstream firms. The benefit of integration is profit sharing between an upstream firm and a downstream firm: vertical externalities are internalized. Vertical integration does not, of course, internalize the horizontal externality. In fact, integration makes the competition more tough. The cost of integration arises from a loss in efficiency and it is assumed to be fixed. We find that the degree of integration is increasing in the size of the downstream market. The profit margin for both input and final good are higher and therefore all three effects work in the same direction

to favour integration. The second result relates to a situation where two upstream firms differ in efficiency. It is intuitively clear that the present model will favour integration by the low-cost firm because it has a higher profit margin and accordingly suffers from greater externalities if unintegrated. Thirdly an industrywide cost increase and higher fixed cost of integration result in a lower degree of integration. All these predictions are robust to the form of competition in the input market.

This analysis also helps us to understand the evolution of the industry structure over time. We predict that in a young market where demand is low and average marginal cost is high (because the learning process is in the beginning) we would see a nonintegrated structure. When the market starts growing and the firms slide down the learning curve the industry becomes more integrated.

Stigler (1951) makes the opposite prediction to us: vertical disintegration is the typical development in growing industries, vertical integration in declining industries. Stigler views production of a final good as a series of distinct functions. Certain functions are subject to decreasing costs. A young market may be too small to support a firm specialized in the function subject to decreasing costs. But when the market expands, the demand for that function becomes sufficient to permit a firm specialized in performing it; the firms spin off the decreasing cost functions and purchase input from the new firm. However, setting up a specialized firm is not the only way to exploit the economies of scale. One integrated firm could produce this input for itself and for the other firms in the industry whatever the size of the market and thus avoid the set-up cost of a new firm. Empirical evidence also suggests that firms frequently integrate as a result of rising, not declining, demand.[1]

Salinger (1988) examines how a vertical merger of successive Cournot oligopolists affects the input and final good price. He finds that a vertical merger does not necessarily

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[1]Porter and Livesay (1971), 132, and Chandler (1977), 490.

increase the prices although a merger leads to foreclosure. The incentives to integrate are the same as in our model. We allow the upstream firms to differ in efficiency and endogenize the integration decision. Ordover, Saloner, and Salop (1990) abstract from double-marginalization by assuming Bertrand competition between equally efficient upstream firms; the input price is driven down to marginal cost. In their model the incentive for integration arises from the assumption that an integrated firm can commit not to supply a rival firm below a certain price. Then the unintegrated upstream firm can raise the input price which will benefit also the integrated firm. Ordover, Saloner, and Salop do not explain why such a commitment is feasible and why the firms have to integrate to be able to commit to such a strategy. In our model foreclosure arises in equilibrium without incredible commitments. In Hart and Tirole (1990) upstream firms set nonlinear prices (essentially Bertrand competition). Low-cost upstream firm has an incentive for integration to restrict competition in the downstream market. Integrated supplier can undercut its high-cost rival slightly, so that the unintegrated firm buys the same total amount as before but now buys from the integrated supplier. This again benefits the integrated firm by raising rival's costs. We show that the predictions this model gives for the degree of integration are qualitatively the same as ours.

Several papers focus on integration decisions in a setting where two upstream firms sell exclusively to two respective downstream firms.[2] Double-marginalization and horizontal externality arise in this setting. If integration induces the rival to be less aggressive, the horizontal externality gives another reason for integration. In other words, when final goods are *strategic substitutes* it is good to be a top dog in the downstream market and integrate to have a lower marginal cost.[3] Both vertical and horizontal externality call for integration. However, when final goods are *strategic complements* the profitable strategy is to be a puppy

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[2]E.g. Vickers (1985), Fershtman and Judd (1987), Bonnano and Vickers (1988), Lin (1988), Gal-Or (1991) and (1992).

[3]We use the terminology of Bulow et al. (1985) and Fudenberg and Tirole (1984).

dog in the downstream market. To eliminate the vertical externality the input price should be equal to marginal cost but to relax the horizontal externality the input price should be higher. Relaxing competition proves to be more important. Tougher competition represents the cost of integration.

The rest of the paper is organized as follows. In Section 2 we introduce our model. Section 3 compares the industry structures and Section 4 derives the equilibrium industry structure. In Section 5 welfare issues are analysed. In Section 6 we compare our model to Hart and Tirole's (1990).

2 The Model

There are two upstream firms, $U1$ and $U2$, producing a homogeneous input, x_i, and two downstream firms, $D1$ and $D2$, producing a homogeneous final good, y_i. Firms have constant returns to scale in the production of input and final good. Furthermore, the final good is produced with a fixed coefficient technology and a unit coefficient for the input x. $U1$ is more efficient in producing the input than $U2$; $c_1 \leq c_2$ where c_i is the marginal cost of input for Ui. The downstream firms are equally efficient and, without loss of generality, we assume that transforming input into the final good is costless.

Demand function for the final good is linear $p_y = a - b(y_1 + y_2)$, where p_y is the price of the final good and a and b are positive constants. We further make the following assumption about the size of the downstream market.

Assumption 1. *The market for the final good is big enough to accommodate both firms. Specifically, a is sufficiently large that the following inequality is satisfied:*

$$a > 11c_2 - 10c_1.$$

Our focus is on the question when the firms will stay independent and when they will vertically integrate. (Horizontal mergers are ruled out by antitrust statutes.) Four structural configurations can emerge: nonintegration, partial integration by the low-cost firm, partial integration by the high-cost firm, and full integration.

The decision timing structure of our model is illustrated in Figure 1.

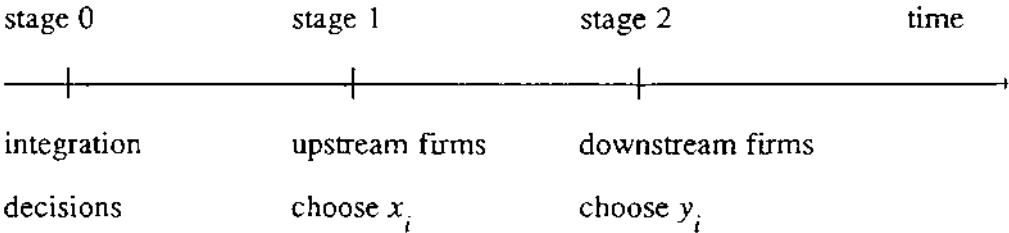| stage 0	stage 1	stage 2	time
integration	upstream firms	downstream firms	
decisions	choose x_i	choose y_i	

Figure 1

At stage 0 the firms decide whether to integrate with a full understanding of the consequences of this decision for the competition in the upstream and downstream markets. We assume that Ui can integrate only with Di. Because the downstream firms are identical, this is not a restrictive assumption. Integration is irreversible. The benefit of integration is profit sharing between an upstream firm and a downstream firm; the vertical externalities are internalized. The cost of integration arising from a loss in efficiency is assumed to be fixed, E. An integrated firm may be less efficient because a non-owning manager has lower incentives to come up with good ideas to reduce production costs or to raise quality because this investment is expropriated by the owner of the firm. Also, there may be a loss in information about the non-owning manager's performance, and therefore less incentive to make improvements. Furthermore, there may be legal costs of the merger.

At stage 1 the upstream firms choose how much input to sell to the downstream firms given the industry structure. In the input market the upstream firms are Cournot duopolists whereas the buyers of input (the downstream firms) take the input price as given. Price taking is just the extreme case where the upstream firms have all the bargaining power. The nature of the vertical externalities would not change if the bargaining power were more equally distributed. The crucial assumption is that pricing is linear. Linear prices apply when the downstream firms could bootleg. When the upstream firms can observe whether or not the downstream firm carries his product, two-part tariffs are optimal contracts (Rey and Tirole (1986)). This alternative assumption about upstream competition is discussed in Section 6. Only unintegrated firms are active in the input market; the integrated firms neither sell nor buy input in the market. Salinger (1988) shows that when the firms are equally efficient in final good production the integrated firm does not sell input to its rival. However, under partial integration the integrated firm would buy input to raise its unintegrated rival's costs. Including these input purchases would only slightly change the tendencies for integration but would unnecessarily complicate the analysis. Therefore we simply assume that the integrated firm will not buy input. (See Halonen (1990) for the overbuying strategy of the integrated firm.) We also assume that exclusive dealing contracts where an upstream firm commits to supplying only one downstream firm are not enforceable.

At stage 2 the downstream firms choose the final good production levels given the input price and the industry structure. The equilibrium in this downstream market generates the derived demand curve for the input at stage 1. Downstream firms behave as Cournot duopolists in the final good market. Cournot competition is justified by our assumptions about the decision timing structure. The downstream market game is played by firms with capacity constraints and the outcome will be Cournot if c_1 and c_2 are high enough.[4]

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[4]Tirole (1988), 215.

The profit function of an integrated firm is:

$$(1) \qquad \pi_i = y_i(p_y - c_i) - E$$

and the profit functions of unintegrated firms are:

$$(2) \qquad \pi_{Ui} = x_i(p_x - c_i); \text{ and}$$

$$(3) \qquad \pi_{Di} = y_i(p_y - p_x)$$

where p_x is the input price.

3 Comparison of Industry Structures

We solve the model by backward induction starting from the last stage. First, we solve for the equilibrium in the downstream market given the input price and the industry structure. The equilibrium in the downstream market generates the derived demand function for the input. Second, we insert this demand function in the upstream firms' profit functions and solve for the equilibrium in the upstream market given the industry structure. Third, we return back to the downstream market to ascertain the subgame perfect equilibrium. Substituting the input price we get the equilibrium in the downstream market in terms of exogenous parameters and the industry structure. (See Appendix for details.) This is how we obtain the profit functions relevant for the stage 0 integration decisions. Under nonintegration (NI) the profits for the upstream and downstream firms are:

$$(4) \qquad \pi_{Ui}^{NI} = 2(a - 2c_i + c_j)^2/27b; \text{ and}$$

$$(5) \qquad \pi_{Di}^{NI} = (2a - c_1 - c_2)^2/81b.$$

Under partial integration by Ui and Di (PIi) the profits are:

(6) $\qquad \pi_{Uj}^{PIi} = (a - 2c_j + c_i)^2/24b;$

(7) $\qquad \pi_{Dj}^{PIi} = (a - 2c_j + c_i)^2/36b;$ and

(8) $\qquad \pi_i^{PIi} = (5a - 7c_i + 2c_j)^2/144b - E.$

And under full integration (FI) we have:

(9) $\qquad \pi_i^{FI} = (a - 2c_i + c_j)^2/9b - E.$

Incentives for integration are driven by three externalities. Consider first a successive monopoly (Spengler (1950)). The vertical externality of *double-marginalization* arises because an unintegrated downstream firm does not take the upstream firm's marginal profit into account when output is increased. Because the downstream firm cares only about its own profit, it tends to make decisions that lead to too low a consumption of input; the industry produces less than the monopoly output. Integration internalizes this externality and enables the industry to earn monopoly profits. The incentive for integration is the greater, the greater is the distortion $(p_x - c_i)$.

Next consider an industry where two upstream firms sell exclusively to two respective downstream firms. Now a *horizontal externality* emerges; the downstream firms destroy profits by competing. Integration does not internalize the horizontal externality since, by assumption, an upstream firm can integrate with only one downstream firm. However, integration has a horizontal effect. High marginal costs (nonintegration) enable the downstream firms to restrict industry output. But given that the rival has high marginal cost the other firm has an incentive to integrate. This will result in lower output by the rival (because reaction functions are downward sloping) which has a positive first order effect on the merged firm's profit. Therefore integration increases the joint profit of the vertical

structure not only because double-marginalization is eliminated but also because integration makes the firm a top dog in the downstream market. In the second vertical merger the horizontal effect is the greater, the larger is the change in the merged firm marginal cost, $(p_x^{PIj} - c_i)$, and the higher that firm's profit margin was originally, $(p_y^{PIj} - p_x^{PIj})$.[5] The latter term describes the benefit from expanding output. If the industry output was already very high (low profit margin) there is little gain from expanding output. In the first vertical merger also the input price for the rival changes. Therefore the horizontal effect depends on how much more favourable the cost change is for the merged firm, $(p_x^{NI} - c_i - p_x^{NI} + p_x^{PIi}) = (p_x^{PIi} - c_i)$, and how profitable the expansion of output is, $(p_y^{NI} - p_x^{NI})$.[6]

Lastly suppose that the upstream firms can sell to both downstream firms. Then an *excessive supply incentive* arises. An unintegrated upstream firm Ui ignores that every unit of input it sells to Dj depresses the final price and reduces Di's profits by $|y_i(\partial p_y/\partial y_j)|$ which is equivalent to Di's profit margin. An unintegrated Ui is selling too much input to Dj compared with the level that would maximize the joint profit of Ui-Di. Excessive supply incentive arises only in nonintegrated industry since under partial integration the integrated firm does not buy input from the unintegrated upstream firm. Table 1 summarizes these three effects.

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[5]Taylor series give the following expression for the horizontal effect:

$$(\partial \pi_i^{PIj}/\partial y_j)(\partial y_j/\partial c_i)(c_i - p_x^{PIj}) + (\partial^2 \pi_i^{PIj}/\partial y_j \partial c_i)(\partial y_j/\partial c_i)(c_i - p_x^{PIj})^2/2 =$$
$$(p_y^{PIj} - p_x^{PIj})(p_x^{PIj} - c_i)/3b + (p_x^{PIj} - c_i)^2/9b$$

[6]The expression for the horizontal effect of the first vertical merger is very complicated and therefore we omit it here. Also what we say in the text is a simplification but serves well to help the intuition.

	NI	PIj
Excessive supply	$(p_y^{NI} - p_x^{NI})$	0
Double-marginalization	$(p_x^{NI} - c_i)$	$(p_x^{PIj} - c_i)$
Horizontal effect	$(p_y^{NI} - p_x^{NI}), (p_x^{PIi} - c_i)$	$(p_y^{PIj} - p_x^{PIj}), (p_x^{PIj} - c_i)$

Table 1

We denote by π_i the vertical structure Ui-Di's joint profit whether it is integrated or not; $\pi_i = \pi_{Ui} + \pi_{Di}$. $\hat{\pi}_i$ stands for the variable profit, that is, profit gross of integration cost. Thus a vertically integrated firm's profit is $\pi_i = \hat{\pi}_i - E$ and an unintegrated firm's profit is $\pi_i = \hat{\pi}_i$. We can easily obtain the following observation about profit levels *ignoring the integration cost.*

Observation 1. $\pi_i^{PIj} < \hat{\pi}_i^{FI} < \pi_i^{NI} < \hat{\pi}_i^{PIi}$

Proof: In the Appendix.

First, integration increases the variable profit of the vertical structure ($\hat{\pi}_i^{PIi} > \pi_i^{NI}$ and $\hat{\pi}_i^{FI} > \pi_i^{PIj}$). The vertical externalities are internalized and in addition integration has a positive horizontal effect as explained earlier. Second, vertical integration imposes a negative externality on the rival ($\pi_i^{PIj} < \pi_i^{NI}$ and $\hat{\pi}_i^{FI} < \hat{\pi}_i^{PIi}$). The merged firm competes more aggressively in the downstream market which makes the rival less aggressive and lowers its profits. Also the upstream unit has lower profits because the demand for input has decreased.

Third, each firm is worse off under full integration than under nonintegration ($\hat{\pi}_i^{FI} < \pi_i^{NI}$). Under full integration *both* firms have lower marginal costs and are more aggressive and, consequently, destroy profits by competing. Although the vertical externalities are internalized the negative horizontal effect dominates and joint profits are lower under full integration than under nonintegration. Figure 2 helps us to understand this result. The solid line shows the monopoly output (assume for a moment that $c_1 = c_2$). The closer the equilibrium is to the solid line, the higher is the producer surplus ignoring the integration costs. We can see that nonintegration has the advantage of restricting output. In fact, it restricts output too much; the industry produces *less* final good than a monopoly would produce. However, output under nonintegration is closer to the monopoly output than output under full integration.

We will now proceed to discuss the comparative statics for the incentives for integration. We can show that:

Proposition 1. *(i) The more efficient firm has a greater incentive to integrate.*

The incentive to integrate is

(ii) decreasing in the degree of integration,

(iii) increasing in the size of the downstream market,

(iv) decreasing in firm's own marginal costs,

(v) increasing in rival's marginal costs, and

(vi) decreasing in the average marginal cost of the industry.

Proof: In the Appendix.

The proof of Proposition 1 is done with the reduced form profit functions (4)-(9) but here we offer intuitive discussion in terms of the profit margins that describe the three effects of a vertical merger (see Table 1).
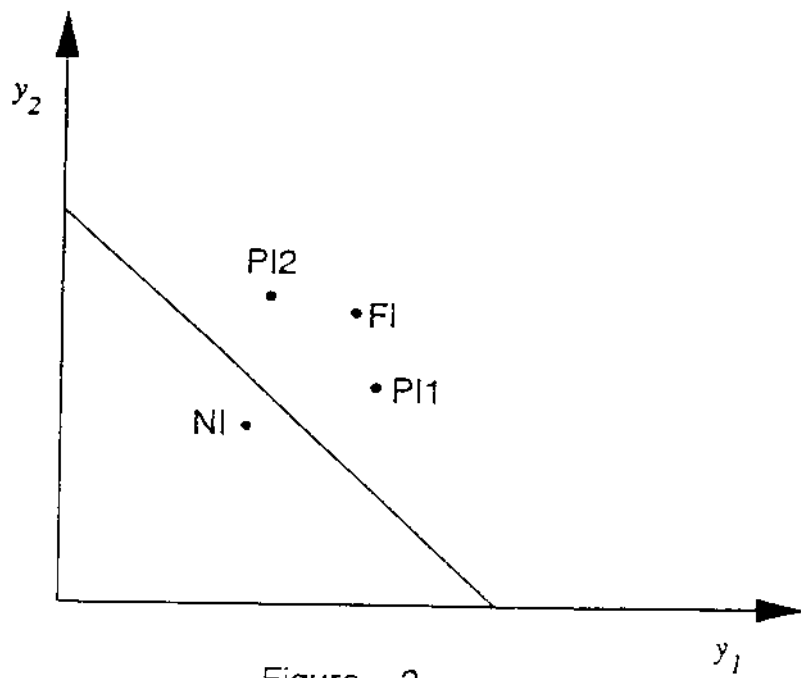
Figure 2

The low-cost upstream firm has a greater incentive to integrate because its profit margin is greater than that of the high-cost firm; both double-marginalization and horizontal effect are greater for the low-cost firm.

The incentive to integrate is greater under nonintegration than under partial integration. Suppose first that the input price is the same under both structures and the upstream firms do not cross supply (that is Ui sells input to Di only). Even in this setting there are diminishing returns to integration. Expansion of output is less profitable if the rival has already expanded because of the horizontal effect. Now take into account that the input price actually decreases after a vertical merger. (We explain the reason for the lower input price in the next paragraph.) Therefore the double-marginalization is smaller under partial integration and this further reduces the incentive for integration. Lastly consider cross supply. Under partial integration there is no excessive supply incentive which gives another reason for lower incentive for integration under partial integration.

A vertical merger results in a lower input price because of two effects on the input market. First, the market power of the remaining supplier increases (in fact, it has now monopoly). Second, the merged firm produces more final good which shifts the residual demand curve of the unintegrated downstream firm back. Accordingly, the demand for the input decreases. These two effects on the input price go in the opposite direction. Salinger (1988) showed that the second effect dominates (and the input price decreases) if and only if less than half of the upstream firms are integrated before the merger. This is why we get this rather surprising effect in duopoly: a vertical merger decreases input price although the integrated firm forecloses the upstream market.

The incentive to integrate is the greater, the greater is the market for final good. All three effects work in the same direction to favour integration. The input price (and the upstream firm's profit margin) is increasing in the size of the market and the final price increases more than the input price and therefore the downstream firm's profit margin increases as well.

The incentive is decreasing in the firm's own marginal cost because a per cent increase in marginal cost increases input price by less than a per cent, which results in a lower profit margin for the upstream firm. Downstream firm's profit margin decreases as well because final price increases less than the input price.

An increase in rival's cost increases both double-marginalization and horizontal effect under partial integration. Under nonintegration double-marginalization is higher, excessive supply incentive is lower and the change in horizontal effect is ambiguous since the upstream firm's profit margin increases and the downstream firm's profit margin decreases. The first effect is dominant and the incentive to integrate is higher. An increase in rival's marginal cost increases input price. It is obvious that input price is increasing in rival's marginal cost when it is selling input. When the rival is integrated (and does not sell input) an increase in its marginal cost decreases its final good production which increases the unintegrated firm's final good production, its demand for input and, accordingly, the input price. An increase in rival's marginal cost increases the downstream firm's profit margin under partial integration but decreases it under nonintegration. Under nonintegration it is not only rival's cost that increases because both downstream firms buy input from both upstream firms. We can also show that an industrywide cost increase lowers the incentive to integrate; the firm's own cost effect is dominant.

4 Equilibrium Industry Structure

Proposition 1 allows us to construct Figure 3 which shows the equilibrium industry structures for various pairs of integration costs (E) and efficiency differences $(c_2 - c_1)$. The figure is drawn for given values of market size and average marginal cost of the industry. The efficiency difference cannot be too big, otherwise $U2$ would not have positive output (which Assumption 1 ensures). The two middle locuses can cross when the market for final good is
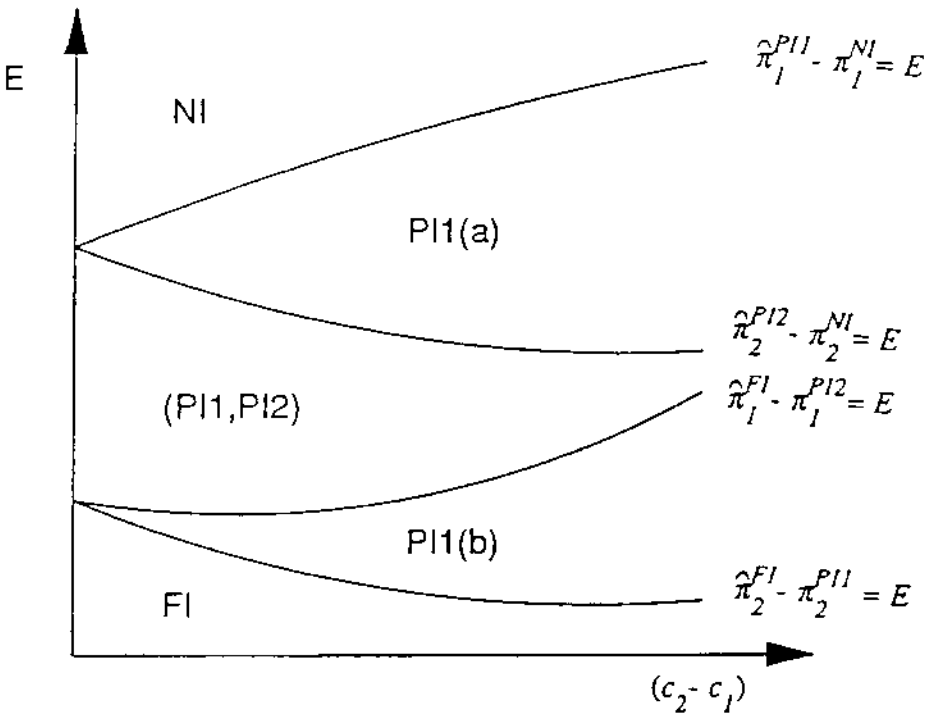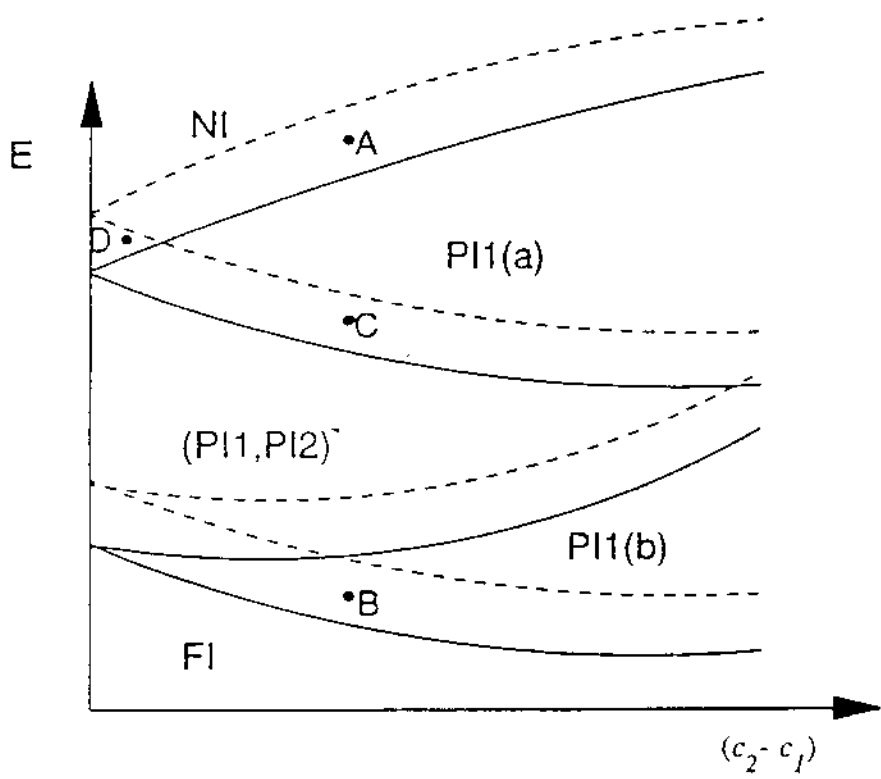
Figure 3

Figure 4

quite small. In the diagram we take the simplest case but Proposition 2 takes into account the possibility that these locuses may cross. We restrict ourselves to pure strategy equilibria.

Proposition 2. *The equilibrium industry structure is*

(i) full integration, if and only if $\hat{\pi}_2^{FI} - \pi_2^{PII} > E$,

(ii) partial integration by the low-cost firm, if $\hat{\pi}_2^{FI} - \pi_2^{PII} < E$ and $\hat{\pi}_1^{FI} - \pi_1^{PI2} > E$, or $\hat{\pi}_2^{PI2} - \pi_2^{NI} < E$ and $\hat{\pi}_1^{PII} - \pi_1^{NI} > E$,

(iii) partial integration by either firm (i.e. there are two equilibria), if and only if $\hat{\pi}_1^{FI} - \pi_1^{PI2} < E$ and $\hat{\pi}_2^{PI2} - \pi_2^{NI} > E$

(iv) nonintegration, if and only if $\hat{\pi}_1^{PII} - \pi_1^{NI} < E$.

To interpret Figure 3, consider the effect of greater integration cost and fix the values of other variables. When the integration cost is high neither firm has an incentive to integrate; nonintegration emerges. When we lower the integration cost it becomes profitable for the low-cost firm to integrate; we have partial integration by the low-cost firm. For even lower integration cost also the high-cost firm can bear the integration cost if the other firm does not bandwagon. Also the low-cost firm integrates if the other firm does not. We have two equilibria: partial integration by either firm. For still lower integration cost the low-cost firm integrates whatever the high-cost firm does and, accordingly, the high-cost firm does not integrate; we have again partial integration by the low-cost firm. When integration cost is very low even the high-cost firm can bear it whatever the rival does and full integration occurs. Note that we have two completely different regions where partial integration by the low-cost firm is the unique equilibrium. In region PI1(*a*) only the low-cost firm can bear the integration cost. And in region PI1(*b*) the high-cost firm can bear the integration cost only if the rival does not bandwagon. But because the low-cost firm integrates whatever the high-cost firm does, the high-cost firm will not integrate.

Next consider the effect of the cost difference keeping integration cost constant (that is, choose a point from the vertical axis). When the firms are equally efficient ($c_1 = c_2$) we are in the vertical axis. For the intermediate values of E only one firm can integrate. Because the firms are identical either firm can integrate in all of this range; there are two equilibria PI1 and PI2. Now increase the efficiency difference. The greater is the efficiency difference, the more likely it is that we end up in a region where only the low-cost firm integrates (PI1). However, starting from *any* point in the vertical axis and increasing the cost difference does not necessarily lead to PI1. For very low values of E we have always full integration, for "very intermediate" values there are always two equilibria, and for very high values of E nonintegration always occurs. The region where PI1 is an equilibrium increases in the efficiency difference and, consequently, the likelihood of PI1 to be an equilibrium is greater.

Then consider an increase in the size of the market for final good. The firms' incentives to integrate are increasing in the size of the market and, accordingly, all the critical locuses in Figure 3 shift upward (see Figure 4). The basic effect is that the degree of integration increases; point A which used to be in the region of nonintegration is now in the region of partial integration and at point B where partial integration occurred the industry becomes fully integrated. By this way we can also select which equilibrium of the multiple ones will emerge in the process of growing market for the final good. Point C which was in the region of partial integration of the low-cost firm is now in the region of two equilibria. Because *UI-DI* was already integrated the equilibrium that will be selected is partial integration by the low-cost firm. However, in our model also partial integration by the high-cost firm can be an equilibrium. Consider point D; the integration cost is fairly high and the firms are almost equally efficient. Originally the industry was nonintegrated. Now, when the size of the market increases we come to the region of two equilibria. Because neither firm was originally integrated it is now possible to have an industry structure where only the high-cost firm integrates.

An increase in the average marginal cost of the industry decreases the firms' incentives to integrate. All the boundaries will shift downwards and have the same effects as lowering the size of the market.

We sum up the comparative static results for the industry structure in the following propositions.

Proposition 3. *The degree of integration is*

(i) increasing in the size of the downstream market,

(ii) decreasing in the integration cost, and

(iii) decreasing in the average marginal cost of the industry.

Proposition 4. *The greater is the efficiency difference between the upstream firms, the more likely is an asymmetric industry structure where only the more efficient firm integrates.*

5 Welfare

In this Section we derive the welfare maximizing industry structure. We use the sum of consumer surplus and producer surplus as a welfare notion (W). There are three sources of deadweight losses in this model: (i) Harberger triangle (final good price is greater than the marginal cost), (ii) production inefficiency (also the high-cost upstream firm has positive output), and (iii) the fixed costs of integration.

In our model a vertical merger always decreases the final good price. The newly integrated firm obtains input at marginal cost which is lower than its Cournot price. This *ceteris paribus* decreases the final good price. However, if there remains an unintegrated firm in the market we have to take into account the effect of the merger on the input price. As was explained earlier a vertical merger lowers the input price which further decreases the final

good price. Consequently, the social gain from integration is the lower final good price which reduces the Harberger triangle.

The production inefficiency is equal to $(c_2 - c_1)x_2$ under NI and PI1 and $(c_2 - c_1)y_2$ under PI2 and FI. The merger by the low-cost firm makes production allocation more efficient and the merger by the high-cost firm makes the production allocation less efficient. We can, however, show that even the merger by the high-cost firm increases variable welfare; the positive effect of the lower final good price outweighs the negative effect of less efficient production allocation. We can show that:

Observation 2. $W^{NI} < \hat{W}^{PI2} < \hat{W}^{PI1} < \hat{W}^{FI}$ *where hats denote the variable (gross of E) values.*

Proof: In the Appendix.

Partial integration by the low-cost firm dominates partial integration by the high-cost firm $(W^{PI2} < W^{PI1})$. The fixed costs of integration are equal under both industry structures but both the final good price and the high-cost firm's output are lower under partial integration by the low-cost firm. Consequently, PI2 is never a social optimum. Variable welfare is the greater, the greater is the degree of integration $(W^{NI} < \hat{W}^{PI1} < \hat{W}^{FI})$, but so is the sum of integration costs. Accordingly, for different parameter values either full integration, partial integration by the low-cost firm or nonintegration can be the social optimum:

Proposition 5. *The welfare maximizing industry structure is*
(i) full integration if and only if $\hat{W}^{FI} - \hat{W}^{PI1} > E$,
(ii) partial integration by the low-cost firm if and only if $\hat{W}^{FI} - \hat{W}^{PI1} < E$ and $\hat{W}^{PI1} - \hat{W}^{NI} > E$, and
(iii) nonintegration if and only if $\hat{W}^{PI1} - \hat{W}^{NI} < E$.

Proof: In the Appendix.

Figure 5 illustrates the critical locuses for both social optimum (solid line) and for Nash equilibrium (broken line). We show in each region the socially optimal structure and the Nash equilibrium structure in brackets if it is different from the socially optimal one. It is straightforward from Figure 5 that:

Proposition 6. *Partial integration by the low-cost upstream firm is less likely than what is welfare maximizing.*

Proof: In the Appendix.

When making its integration decision the firm ignores the negative externality for rival and the positive externality for consumers. It turns out that the first merger is very beneficial for the consumers and is not very harmful for the rival; the first merger occurs too late (in terms of growing market). The second merger does not offer a much lower price for the consumers but harms the rival a lot; the second merger emerges too early.

6 Nonlinear Prices

To conclude, we compare our results to those of Hart and Tirole's (1990) Model 1 (hereafter H-T) which has nonlinear prices in the input market (essentially Bertrand competition). Both models have Cournot competition in the final good market. In both models the benefit of integration is profit sharing and there is a fixed cost of integration. We follow Sutton (1991) in using Cournot and Bertrand models as special examples within a general class of models which differ in toughness of price competition. Bertrand has the most severe price competition where only the most efficient firm can survive. Cournot corresponds
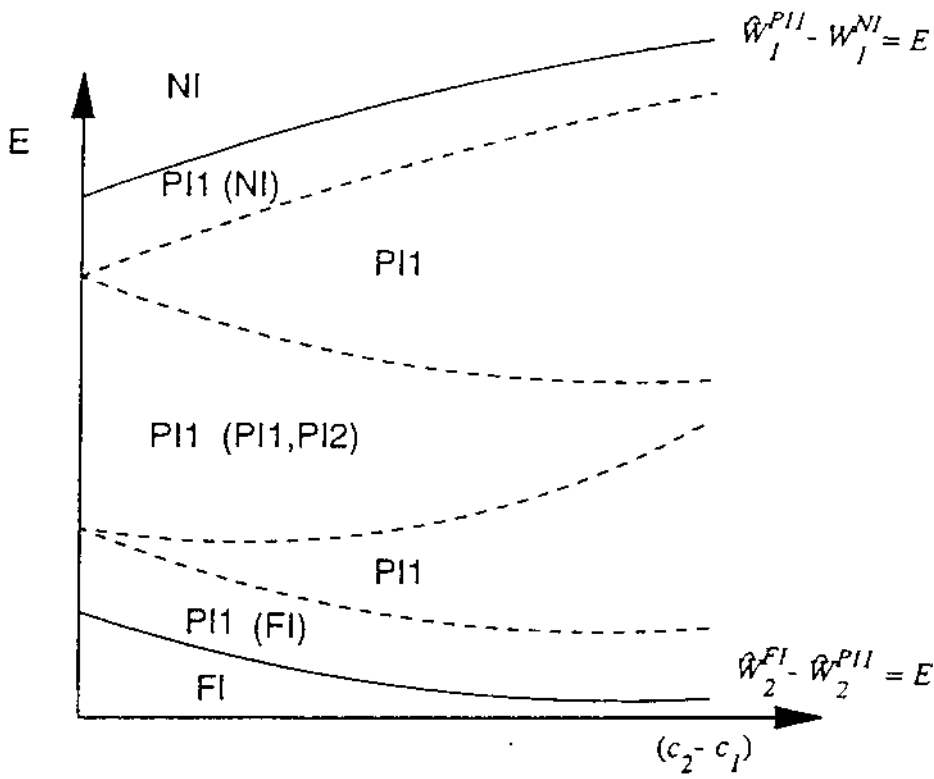
NI

E

PI1 (NI)

PI1

$\hat{W}_1^{PII} - W_1^{NI} = E$

PI1 (PI1,PI2)

PI1

PI1 (FI)

FI

$\hat{W}_2^{FI} - \hat{W}_2^{PII} = E$

$(c_2 - c_1)$
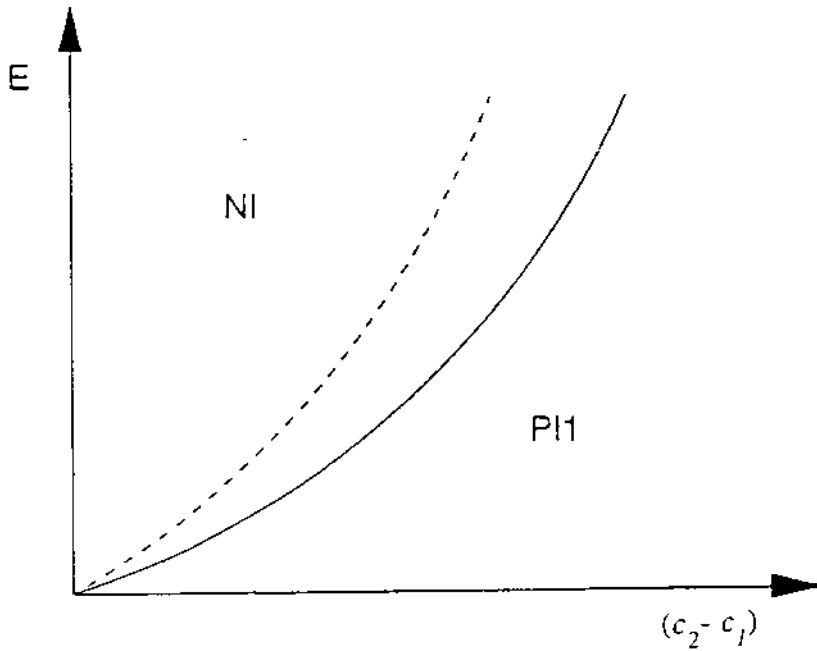
Figure   5

E

NI

PI1

$(c_2 - c_1)$

Figure   6

to more relaxed price competition where also the less efficient firm has a positive market share. Our aim was to have predictions for the degree of integration that are robust to the toughness of price competition in the input market.

H-T do not provide comparative static analysis for the industry structure but it is a simple matter to do that for linear demand. Figure 6 illustrates the equilibrium industry structures in their model. Competition in the input market is so severe that the high-cost firm cannot sell any input. Even if it integrated with a downstream firm, its downstream unit would buy all its input from the low-cost firm. Accordingly, high-cost firm does not gain anything from integration. Its incentive to integrate would be on the horizontal axis in Figure 6. The upward sloping critical locus gives the low-cost firm's incentive to integrate. When the firms have equal marginal costs neither firm has an incentive to integrate. The incentive is increasing in the efficiency difference. Low-cost upstream firm has an incentive for integration to restrict competition in the downstream market. If the low-cost supplier is unintegrated it cannot commit to supply one downstream firm only; excessive supply incentive arises. Integrated supplier internalizes this externality and can undercut its high-cost rival slightly, so that the unintegrated firm buys the same total amount as before but now buys from the integrated supplier. This increases rival's costs which has a positive horizontal effect on the integrated firm's profits. The equilibrium industry structure is either nonintegration or partial integration by the low-cost firm.

A greater market size or a lower average marginal cost will bend the critical locus upwards (broken line). We find that the comparative static results for the degree of integration are the same as in our model. The degree of integration is the greater (*i*) the greater the size of the downstream market, (*ii*) the lower the integration cost, and (*iii*) the lower the average marginal cost of the industry. Also, the greater is the efficiency difference, the more likely is an asymmetric industry structure where only the low-cost firm is integrated. The main point of our paper is that qualitatively the same pattern of integration emerges whether there is Cournot or Bertrand competition in the input market.

Welfare results differ. In H-T nonintegration is always the social optimum because integration restricts output and increases fixed costs of integration. In our model also an industry structure with vertical integration can be welfare maximizing because integration increases output although it also increases the fixed costs. H-T find excessive integration in Nash equilibrium and we find a less asymmetric industry structure than what is welfare maximizing.

Appendix

We solve the nonintegrated case as an example. Full integration and partial integration follow in a similar manner. The profit functions of the firms are:

(A.1)
$$\pi_{Di} = y_i(a - by_i - by_j - p_x) \qquad i,j = 1,2 \quad i \ne j$$

(A.2)
$$\pi_{Ui} = x_i(p_x - c_i) \qquad i = 1,2$$

To solve for the subgame perfect equilibrium, we work backwards and solve first for the equilibrium in the downstream market given the input price p_x. The equilibrium quantities are:

(A.3)
$$y_i = (a - p_x)/3b$$

Substituting the upstream market clearing condition

(A.4)
$$y_1 + y_2 = x_1 + x_2$$

into (A.3) we can solve for the inverse demand for input; this gives the price at which the unintegrated downstream firms are willing to buy the input quantity supplied by the unintegrated upstream firms:

(A.5)
$$p_x = [2a - 3b(x_1 + x_2)]/2.$$

Next we turn to examine the first stage. Inserting p_x from (A.5) to (A.2) we get the unintegrated upstream firm's profit expressed in terms of x_i's only:

(A.6)
$$\pi_{Ui} = x_i\{[2a - 3b(x_1 + x_2)]/2 - c_i\}$$

The equilibrium quantities in the upstream market are thus:

(A.7)
$$x_i = 2(a - 2c_i + c_j)/9b.$$

The equilibrium price is:

(A.8)
$$p_x = (a + c_1 + c_2)/3.$$

Finally, we can return back to the downstream market to ascertain the subgame perfect equilibrium in terms of the exogenous parameters only. When we insert (A.8) into (A.3) we get the equilibrium quantities:

(A.9)
$$y_i = (2a - c_1 - c_2)/9b$$

The equilibrium price is:

(A.10)
$$p_y = [5a + 2(c_1 + c_2)]/9$$

And the equilibrium profits of the firms are:

(A.11)
$$\pi_{Di} = (2a - c_1 - c_2)^2/81b$$

(A.12)
$$\pi_{Ui} = 2(a - 2c_i + c_j)^2/27b$$

Under partial integration by Ui and Di we can solve for the equilibrium in a similar manner. The equilibrium quantities, prices and profits are:

(A.13)
$$x_j = y_j = (a - 2c_j + c_i)/6b$$

(A.14)
$$y_i = (5a - 7c_i + 2c_j)/12b$$

(A.15)
$$p_x = (a + 2c_j + c_i)/4$$

(A.16)
$$p_y = (5a + 2c_j + 5c_i)/12$$

(A.17)
$$\pi_{Uj} = (a - 2c_j + c_i)^2/24b$$

(A.18)
$$\pi_{Dj} = (a - 2c_j + c_i)^2/36b$$

(A.19)
$$\pi_i = (5a - 7c_i + 2c_j)^2/144b - E$$

And under full integration:

(A.20)
$$y_i = (a - 2c_i + c_j)/3b$$

(A.21)
$$p_y = (a + c_i + c_j)/3$$

(A.22)
$$\pi_i = (a - 2c_i + c_j)^2/9b - E$$

Proof of Observation 1:

Step 1: $\pi_i^{PIj} < \hat{\pi}_i^{FI}$

(A.23)
$$(a - 2c_i + c_j)^2/24b + (a - 2c_i + c_j)^2/36b < (a - 2c_i + c_j)^2/9b$$

Simplifying

(A.24)
$$3(a - 2c_i + c_j)^2/72b > 0$$

Step 2: $\hat{\pi}_i^{FI} < \pi_i^{NI}$

(A.25)
$$(a - 2c_i + c_j)^2/9b < 2(a - 2c_i + c_j)^2/27b + (2a - c_1 - c_2)^2/81b$$

(*iii*) the incentive is increasing in the size of the downstream market

(A.38) $\partial(\hat{\pi}_i^{PIi} - \pi_i^{NI})/\partial a = (130a - 182c_i + 52c_j)/1296b > 0$

(A.39) $\partial(\hat{\pi}_i^{FI} - \pi_i^{PIj})/\partial a = 2(a - 2c_i + c_j)/24b > 0$

(*iv*) the incentive is decreasing in firm's own marginal cost

(A.40) $\partial(\hat{\pi}_i^{PIi} - \pi_i^{NI})/\partial c_i = -(182a - 82c_i - 100c_j)/1296b < 0$

(A.41) $\partial(\hat{\pi}_i^{FI} - \pi_i^{PIj})/\partial c_i = -4(a - 2c_i + c_j)/24b < 0$

(*v*) the incentive is increasing in rival's marginal cost

(A.42) $\partial(\hat{\pi}_i^{PIi} - \pi_i^{NI})/\partial c_j = (52a - 100c_i - 152c_j)/1296b > 0$

(A.43) $\partial(\hat{\pi}_i^{FI} - \pi_i^{PIj})/\partial c_j = 2(a - 2c_i + c_j)/24b > 0$

(*vi*) the incentive is decreasing in the average marginal cost of the industry

(A.44) $\sum_{j=1}^{2} \partial(\hat{\pi}_i^{PIi} - \pi_i^{NI})/\partial c_j = -(130a + 18c_i + 52c_j)/1296b < 0$

(A.45) $\sum_{j=1}^{2} \partial(\hat{\pi}_i^{FI} - \pi_i^{PIj})/\partial c_j = -2(a - 2c_i + c_j)/24b < 0$

(*i*) the more efficient firm has a greater incentive to integrate

When the firms have equal marginal costs the incentives to integrate are equal (equations (A.31) and (A.32)). Now increase c_2. (A.40)-(A.43) show that *U1-D1*'s incentive increases and *U2-D2*'s incentive decreases. Therefore *U1-D1*'s incentive is greater than *U2-D2*'s when $c_1 < c_2$.

<div align="right">Q.E.D.</div>

Simplifying

(A.26) $[(2a - c_1 - c_2) - \sqrt{3}(a - 2c_i + c_j)]/9\sqrt{b} > 0$

(A.27) $\iff a > [(2\sqrt{3} - 1)c_j - (\sqrt{3} + 1)c_i]/(2 - \sqrt{3})$

Step 3: $\pi_i^{NI} < \hat{\pi}_i^{PIi}$

(A.28) $2(a - 2c_i + c_j)^2/27b + (2a - c_1 - c_2)^2/81b < (5a - 7c_i + 2c_j)^2/144b$

Simplifying

(A.29) $[(a-c_j)(65a + 117c_j - 182c_i) + 41(c_i-c_j)^2]/1296b > 0$

(A.30) $\iff a > (182/65)c_i - (117/65)c_j$

Assumption 1 guarantees that (A.24), (A.27) and (A.30) are satisfied.

Q.E.D.

Proof of Proposition 1:

The incentives to integrate are:

(A.31) $\hat{\pi}_i^{PIi} - \pi_i^{NI} = [(a-c_j)(65a - 182c_i + 117c_j) + 41(c_j-c_i)^2]/1296b$

(A.32) $\hat{\pi}_i^{FI} - \pi_i^{PIj} = (a - 2c_i + c_j)^2/24b$

(ii) the incentive is decreasing in the degree of integration: $\hat{\pi}_i^{PIi} - \pi_i^{NI} > \hat{\pi}_i^{FI} - \pi_i^{PIj}$

(A.33) $(5a - 7c_i + 2c_j)^2/144b - 2(a - 2c_i + c_j)^2/27b - (2a - c_1 + c_2)^2/81b >$

$(a - 2c_i + c_j)^2/24b$

Simplifying

(A.34) $[(a-c_i)(11a+45c_i-56c_j) - 130(c_j-c_i)^2]/1296b > 0$

(A.34) holds if the following conditions are satisfied:

(A.35) $11a+45c_1-56c_2 > 0 \iff a > (56/11)c_2 - (45/11)c_1$

(A.36) $11a+45c_1-56c_2 > 65(c_2-c_1) \iff a > (121/11)c_2 - (110/11)c_1$

(A.37) $a-c_1 > 2(c_2-c_1) \iff a > 2c_2 - c_1$

Assumption 1 guarantees that these conditions are satisfied.

Proof of Observation 2:

Step 1: $W^{NI} < \hat{W}^{PI2}$

(A.46) $\qquad 2(2a - c_1 - c_2)^2/81b + 2(a - 2c_1 + c_2)^2/27b + 2(a - 2c_2 + c_1)^2/27b$

$\qquad\qquad + 2(2a - c_1 - c_2)^2/81b < (7a - 2c_1 - 5c_2)^2/288b + 5(a - 2c_1 + c_2)^2/72b$

$\qquad\qquad + (5a - 7c_2 + 2c_1)^2/144b$

Simplifying

(A.47) $\qquad [(a-c_1)(175a+459c_1-634c_2) + 199(c_2-c_1)^2]/2592b > 0$

(A.48) $\qquad <= \ a > (634/175)c_2 - (459/175)c_1$

Step 2: $\hat{W}^{PI2} < \hat{W}^{PII}$

Step 2(a): $CS^{PII} > CS^{PI2} \ <=> \ p_y^{PII} < p_y^{PI2}$

where CS is consumer surplus

(A.49) $\qquad (5a + 2c_2 + 5c_1)/12 < (5a + 2c_1 + 5c_2)/12$

(A.50) $\qquad <=> \ c_1 < c_2$

Step 2(b): $\pi_1^{PII} + \pi_2^{PII} > \pi_2^{PI2} + \pi_1^{PI2}$

(A.51) $\qquad (5a - 7c_1 + 2c_2)^2/144b + 5(a - 2c_2 + c_1)^2/72b >$

$\qquad\qquad (5a - 7c_2 + 2c_1)^2/144b + 5(a - 2c_1 + c_2)^2/72b$

(A.52) $\qquad <=> \ (a-c_1) > (a-c_2)$

Step 3: $\hat{W}^{PII} < \hat{W}^{FI}$

(A.53) $\qquad (7a - 5c_1 - 2c_2)^2/288b + (5a - 7c_1 + 2c_2)^2/144b + 5(a - 2c_2 + c_1)^2/72b$

$\qquad\qquad < (2a - c_1 - c_2)^2/18b + (a - 2c_1 + c_2)^2/9b + (a - 2c_2 + c_1)^2/9b$

Simplifying

(A.54) $\qquad [(a-c_1)(9a+51c_1-60c_2) + 84(c_2-c_1)^2]/2592b > 0$

(A.55) $\qquad <= \ a > (60/9)c_2 - (51/9)c_1$

Assumption 1 ensures that (A.48), (A.50), (A.52) and (A.55) are satisfied.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ Q.E.D.

Proof of Proposition 5:

$\hat{W}^{FI} - \hat{W}^{PII} < \hat{W}^{PII} - \hat{W}^{NI}$

(A.56) $\Longleftrightarrow CS^{FI} + \pi_1^{FI} + \pi_2^{FI} + CS^{NI} + \pi_1^{NI} + \pi_2^{NI} < 2CS^{PII} + 2\pi_1^{PII} + 2\pi_2^{PII}$

(A.57) $\Longleftrightarrow (2a - c_1 - c_2)^2/18b + (a - 2c_1 + c_2)^2/9b + (a - 2c_2 + c_1)^2/9b +$

$2(2a - c_1 - c_2)^2/81b + 2(a - 2c_1 + c_2)^2/27b + 2(a - 2c_2 + c_1)^2/27b +$

$2(2a - c_1 - c_2)^2/81b < 2(7a - 5c_1 - 2c_2)^2/288b + 2(5a - 7c_1 + 2c_2)^2/144b +$

$10(a - 2c_2 + c_1)^2/72b$

Simplifying

(A.58) $[47(a - c_1)^2 + 316(c_2 - c_1)(a - c_2) + 96(c_2 - c_1)(a - 2c_2 + c_1)]/2592b > 0$

$Q.E.D$

Proof of Proposition 6:

Step 1: $\hat{W}^{FI} - \hat{W}^{PII} < \hat{\pi}_2^{FI} - \pi_2^{PII}$

(A.59) $\Longleftrightarrow CS^{FI} + \hat{\pi}_1^{FI} + \hat{\pi}_2^{FI} - CS^{PII} - \hat{\pi}_1^{PII} - \pi_2^{PII} < \hat{\pi}_2^{FI} - \pi_2^{PII}$

(A.60) $\Longleftrightarrow CS^{FI} + \hat{\pi}_1^{FI} < CS^{PII} + \hat{\pi}_1^{PII}$

(A.61) $\Longleftrightarrow (2a - c_1 - c_2)^2/81b + (a - 2c_1 + c_2)^2/9b$

$< (7a - 5c_1 - 2c_2)^2/288b + (5a - 7c_1 + 2c_2)^2/144b$

Simplifying

(A.62) $[(a-c_1)(475a-135c_1-340c_2) - 212(c_2-c_1)^2]/2592b > 0$

True if the following conditions are satisfied:

(A.63) $475a-135c_1-340c_2 > 0 \Longleftrightarrow a > (135/475)c_1 + (340/475)c_2$

(A.64) $475a-135c_1-340c_2 > 212(c_2-c_1) \Longleftrightarrow a > (552/475)c_2 - (77/475)c_1$

(A.65) $a-c_1 > c_2-c_1 \Longleftrightarrow a > c_2$

Assumption 1 guarantees that these conditions are satisfied.

Step 2: $\hat{W}^{PII} - W^{NI} > \hat{\pi}_1^{PII} - \pi_1^{NI}$

(A.66) $\Longleftrightarrow CS^{PII} + \hat{\pi}_1^{PII} + \pi_2^{PII} - CS^{NI} - \pi_1^{NI} - \pi_2^{NI} > \hat{\pi}_1^{PII} - \pi_1^{NI}$

(A.67) $\Longleftrightarrow CS^{PII} + \pi_2^{PII} > CS^{NI} + \pi_2^{NI}$

(A.68) $\Longleftrightarrow (7a - 5c_1 - 2c_2)^2/288b + 5(a + c_1 - 2c_2)^2/72b >$

$2(2a - c_1 - c_2)^2/81b + 2(a + c_1 - 2c_2)^2/27b + (2a - c_1 - c_2)^2/81b$

(A.69) $\Longleftrightarrow [45(a-c_1)^2 + 108(c_2-c_1)(a-c_2) + 72(c_2-c_1)(a-c_1)]/2592b > 0$

Q.E.D.

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